Sovereign and bank solvency risks

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Economics

Master's Thesis

September 2017

HELSINGIN YLIOPISTO
HELSINGFORS UNIVERSITET
UNIVERSITY OF HELSINKI

Tiedekunta/Osasto – Fakultet/Sektion – Faculty	Laitos – Institution – Department
Faculty of Social Sciences	Department of Political and Economic Studies

Tekijä – Författare – Author
Juho Martti Peltonen

Työn nimi – Arbetets titel – Title
Sovereign and bank solvency risks

Oppiaine – Läroämne – Subject
Economics

Työn laji – Arbetets art – Level	Aika – Datum – Month and year	Sivumäärä – Sidoantal – Number of pages
Master's Thesis	17.9.2017	76

Tiivistelmä – Referat – Abstract

The Euro crisis 2009–2012 made sovereign debt a significant problem in many European countries, which was a significant change to the prevailed thought that the sovereign debt of developed country is a safe asset. The increase in sovereign default risk was recognized to be in close connection with the banking sector credit risk. This thesis explains the relationship of sovereign and bank credit risk, a phenomenon called "the feedback loop".

The feedback loop occurs between the sovereign and banks when the deterioration of sovereign creditworthiness reduces the market value of banks' holdings of sovereign debt. The value reduction decreases the solvency of banks and in the worst case leads to insolvency, which requires the sovereign to bail out the banks. This in turn induces negative feedback to the sovereign increasing risk and reducing the market value of the sovereign debt even further, creating a self-perpetuating loop.

The formal mathematical model of the feedback loop is presented in this thesis. First a one-country model, which is then extended to a two-country model. Using these models, the channels linking the sovereign and banks together, as well as the factors affecting the occurrence of the feedback loop can be examined in detail.

Further, I create a model of a banking union, based on the two-country model. The proposed model shows how the banking union can prevent the feedback loop. The banking union model is also used to study issues affecting the design of the banking union, like the limits of its costs and resources.

The analysis reveals, that when banks' equity and a sovereign fiscal surplus are low, and banks are exposed to sovereign debt, a drop in a value of sovereign debt leads to a feedback loop. The two-country model shows that the feedback loop can also be the result of a price drop of foreign debt and that risk contagion can occur between countries. Banks may also be exposed to part of the sovereign debt risk without direct holdings, through their holdings of the foreign sovereign debt. In addition, the role of banking union, which is responsible for bailing out banks instead of the sovereign, is studied with the two-country model. The feedback loop can be prevented by requiring banks to hold sufficient amounts of equity relative to their sovereign exposures. Furthermore, the banking union removes the link between banks and the sovereign, preventing the feedback loop from starting. However, the banking unions success is dependent of the design of the union, and may in cases of too small resources lead to worse situation in member states than without the union.

(JEL H63, G21, G01)

Avainsanat – Nyckelord – Keywords
Sovereign debt, Feedback loop, Bailout, Banking union, Bank insolvency, Sovereign default, Euro crisis

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1 Introduction

This thesis aims to create deeper understanding about the vitally important interplay between a sovereign debt and a bank credit risk. Before financial crisis in 2007–2008, the sovereign debt of developed countries was widely considered a safe asset for investors. The financial crisis and especially Euro crisis that followed in 2009-2012 changed this understanding, by making the sovereign debt a significant problem in many European countries. This development can be illustrated with figure 1, that shows the long-term government bond yields in some Euro countries. From 2008 onwards, the graph reveals significant divergence between the so called Eurozone core and periphery countries. This divergence reflects significant change in the investors' perception of the risk related to these countries.

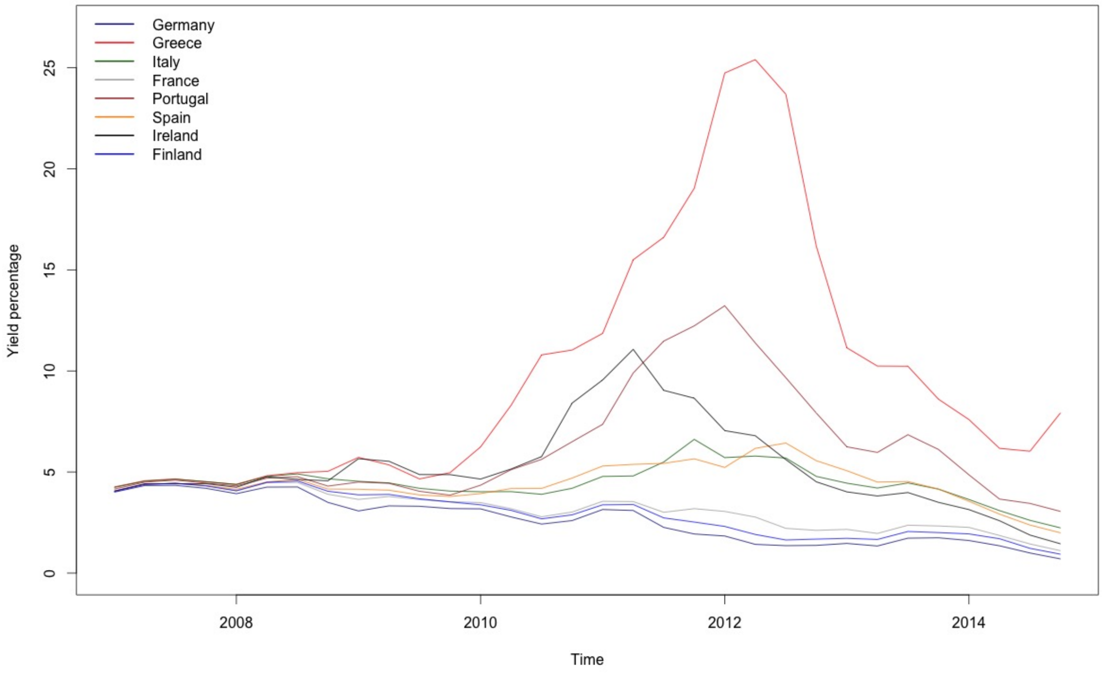

Figure 1: 10 year government bond yields in some Eurozone countries 2007–2015, data from Fred[1], graph follows the one in Battistini et al. (2014).

[1]Federal Reserve Bank of St. Louis, FRED Economic Data, http://fred.stlouisfed.org

One of the countries that suffered a large increase in bond yields, and related debt risk, was Ireland. Figure 2 shows the development of Irish sovereign CDS (credit default swap) and Irish bank CDS prices. The CDS is used to hedge a credit default and its price is then an indicator of the risk level of the underlying debt. The graph shows that at the date when Irish government announced to guarantee the liabilities of Irish banks, bailed out the banks, the risk of banks drops, but the sovereign risk jumps immediately. After the bailout event two risks are developing very similarly. This example of Ireland indicates a close connection between sovereign and bank credit risks as a reason for Euro crisis.

Figure 2: The sovereign CDS (credit default swap) and bank CDS for Ireland in 2007–2010, graph from Acharya et al. (2014). [2]

This thesis explains the relationship of sovereign and bank credit risk. This relationship is called a feedback loop, or a diabolic loop, or a doom loop. The feedback loop occurs when the deterioration of sovereign creditworthiness reduces the market value of sovereign bonds that banks hold as assets, which may lead to bank insolvency and require sovereign bailout. This induces negative feedback to the sovereign increasing risk and reducing

[2]Acharya et al. (2014): "The bank CDS is computed as the equally-weighted average of bank CDS for banks headquartered in Ireland (Allied Irish Bank, Anglo Irish Bank, Bank of Ireland, and Irish Life and Permanent). The data are from Datastream."

bond prices even further, creating a self-perpetuating loop. Home bias, banks holding excess amount of domestic sovereign debt, existence of a bailout option and free capital movements are ingredients enabling the feedback loop.

The formal mathematical model of the feedback loop is presented, first as a one-country model that follows the work of Brunnermeier et al. (2016), which is then extended to a two-country version. The models reveal that when banks' equity and a sovereign fiscal surplus are low, and banks hold substantial amount of sovereign debt, a drop in a value of sovereign debt leads to a feedback loop equilibrium. The two-country model reveals that also foreign sovereign debt is a risky asset to banks, and feedback loop contagion can occur between countries. The two-country model also shows that risk related to the investment portfolio of a bank depends on the portfolios of banks in the other country, and reveals the complexity of cross dependencies between countries and their banking sectors. Finally, a model of banking union is presented. The idea in the banking union is that it performs the bank bailouts, and thus it can break the link from banks to sovereign and prevent the feedback loop. I order to succeed in this task the banking union has to possess enough resources.

The structure of the thesis is following. Chapter 2 presents the background theory of the feedback loop and the banking union. Chapter 3 contains a literature review of the feedback loop models. The one-country model is presented in Chapter 4 and two-country extension in 5 while Chapter 6 presents the banking union model. Chapter 7 contains analysis based on the models and Chapter 8 concludes the thesis.

2 Theory of "feedback loop"

This chapter provides a review of the feedback loop theories and background in the academic literature. The basis of the feedback loop lies in some well-known facts like the fragility of sovereign debt and banking, and some preconditions, like the home bias and the theory of efficient bailouts. These fundamentals are presented first and the feedback loop theory itself after them in this chapter. Finally, some empirical research and background information is presented about how the relationship between sovereign and bank credit risk was involved in Euro crisis.

2.1 Fragility of sovereign debt and banking

The feedback loop between the sovereign and bank credit risk is formed between the sovereign credit risk and private banks balance sheets, both of which have the fundamental elements of fragility on their own as well (Cooper and Nikolov, 2013). The fragility of sovereign debt is famously presented by Calvo (1988) and it arises due to a strategic complementarity between the buyers of government bonds and the government default decision, which can lead to self-fulfilling prophecy. This means that if lender has concerns about the government's ability to repay the sovereign debt, it requires a higher interest rate, while the higher interest rate weakens the government's solvency and validates the pessimistic expectations.

The probability of government defaulting its debt is often seen as a problem for external debt while domestic sovereign debt is considered safe. The idea is that sovereigns cannot become over-indebted domestically in their own currency, because they can always inflate the debt away. Reinhart and Rogoff (2011) show that historically the domestic defaults cannot be called rare events, even though they are less common than external defaults. The reason why domestic debt may be defaulted is the costs and distortions that inflation creates.

Pozzolo et al. (2016) argue that default of Argentina in 2002 or the partial default of Greece in 2011 serve as examples that verify the true default risk included in domestic sovereign debt, even though the probability of default is low. Also, studies like Acharya et al. (2014) or Battistini et al. (2014) show that during Euro crisis investors started to take the sovereign default risk more seriously. In the feedback loop context, the default risk contained in the domestic debt is important aspect, since the loop occurs between the sovereign and its domestic creditors, specifically banks.

The most famous presentation of banking instability is probably Diamond and Dybvig (1983), where the authors present the so called bank run. Like the fragility of government debt in Calvo (1988), the bank run is also based on the idea of a self-fulfilling prophesy. Now the depositors fear that the bank may fail and thus they rush to withdraw their deposits. Because of the sudden withdrawals, the bank may be forced to liquidate many of its assets at a loss, which eventually is the reason for the banks insolvency and failure. To avoid bank runs government can provide guarantees to the deposit, which effectively prevents bank runs from happening due to the idea that the government is always able to gather as much resources as needed to finance the losses to depositors (Diamond and Dybvig, 1983). However, government guarantees link the sovereign and private banks together in a way where the bank risk may transfer to the sovereign risk, thus creating one side of the feedback loop.

2.2 Home bias

Investment portfolio allocation, where an investor overwhelmingly prefers to hold domestic equity assets is referred to as "home bias". This type of allocation is biased because investors hold oversized shares of domestic assets despite the potential gains and risk level reduction from international diversification. (Tesar and Werner 1995.)

Private investors have various reasons for home bias, including irrational individual tendencies (Tesar and Werner, 1995). Banks however, differ from private investors, since they are reliant on national policies and international financial regulations. Especially important is the home bias related to the banks holdings of domestic government bonds. It is obvious that the larger the share of domestic sovereign debt in the banks' portfolio, the more exposed it is to the country specific risk of sovereign debt (Acharya et al., 2014). The home bias in investments in government bonds is one of the key preconditions for the occurrence of the feedback loop.

Many studies have shown the statistics of severe home bias during Euro crisis. According to Acharya et al. (2014), at 2010 in Eurozone almost 70 % of bonds held by banks were issued by the country in which a bank was headquartered. Similar results can be seen from the European bank stress test data in 2011, presented by Uhlig (2014), where the large group of countries exceed the 50 % home share of the sovereign exposure. Battistini et al. (2014) notice that increase in home bias during crisis is at least partially due to banks replacing their foreign debt holdings with domestic ones.

There are several reasons why banks allocate their investments heavily on domestic sovereign debt, especially during crisis. The first explanation is set of policies that Chari et al. (2014) call as "financial repression". This means that government engages policies that force banks to hold government debt, when the government faces exceptionally high fiscal needs, e.g. during financial crises or wartime, or when foreign lending suddenly stops. While financial repression contains legal methods to oblige banks to buying government debt, a little less compelling type of government behavior is called “moral suasion”, e.g. by Battistini et al. (2014) or Pozzolo et al. (2016). Moral suasion means that the government pressures banks to buying more bonds, especially if banks have previously been saved by the government using taxpayers' money. Uhlig (2014) complements financial repression or moral suasion theories with a political economy explanation, where banks' holdings of domestic sovereign debt serve as a commitment tool for the government, since defaulting domestically held debt is politically more difficult as it hurts domestic banks and savers.

Acharya and Steffen (2015) suggest an alternative explanation to home bias called the carry-trade effect. The banks invest in risky and thus high-yield sovereign debt during crisis and try to exploit the larger price swings of these sovereign bonds. In Eurozone, in the most extreme case, the banks may even borrow the money that they invest in domestic bonds from the European Central Bank. However, as Pozzolo et al. (2016) point out the carry-trade motive only justifies bias towards high risk government bonds, not exactly home bias. Nevertheless, the carry-trade explanation can be complemented with an argument that banks cannot hedge the domestic sovereign debt risk, because if sovereign really does default, the banks run out of business regardless of their sovereign debt portfolio allocations. Thus, the banks have “nothing to lose” and a rational choice is trying to benefit from the carry-trade (Pozzolo et al., 2016).

During the Euro crisis banks faced a real threat that Euro will collapse. If this would have happened the liabilities of banks, as the holdings of domestic sovereign debt, would have been redenominated into new national currencies. This means that domestic banks had “comparative advantage” against the redenomination risk compared with foreign banks, which may have been Eurozone specific explanation of the home bias during the crisis (Battistini et al., 2014).

Finally, regardless of the crisis, the financial regulation may make domestic sovereign debt appealing to domestic banks. In International Basel II accord the risk weight of the sovereign depends on the credit assessment classification, but the accord also includes a possibility to have national discretion about the risk weight between domestic banks and the sovereign (Basel Committee on Banking Supervision, 2006). The lower the risk weight of an asset the less requirements for capital is set to banks holding these assets. There are significant differences in the application of the Basel rules across jurisdictions, in the European Union authorities have set a zero risk weight to sovereign exposures denominated and funded in the currency of the corresponding member state (BIS Monetary and Economic Department, 2013). Also the European Systemic Risk Board (2015) notifies in their report that the prudential regulations treat the sovereign exposures favorably, e.g. by setting low capital requirements, categorizing some government bonds as highly liquid assets, or not limiting the large exposures. Thus, the favorable regulation, may increase the home bias even when there is no crisis.

2.3 Bank bailouts

A bailout channel is essential mechanism in feedback loop connecting banks' credit risk to sovereign. In practice bailout means that government recapitalizes insolvent banks, or offers to insure their liabilities. The bailout has provoked lots of public and academic discussion in questions like, whether sovereign should bailout banks at all, under which circumstances the bailout is beneficial, or what kind of bailout policy leads to the best outcome.

Typically, the government bails out banks to protect depositors, borrowers or bankers, and rationalities why all these groups merit protection have been presented (Diamond and Rajan, 2002). One common argument is that bailout is necessary to avoid a complete meltdown of the financial sector, which could lead to substantial effects on real economy (Bianchi, 2016). Classic example is Diamond-Dybvig bank run model, where the government deposit guarantees effectively prevent bank runs (Diamond and Dybvig, 1983).

Bailout can improve the efficiency of resource allocation from a social point of view, or have stabilizing effects on the economy. This is because, if all risk would be concentrated in the private sector, it would be more cautious and offers less financial services. This under provision of resources leads to welfare losses. Also, in time of crisis, private investors would be less eager to run at the first sign of trouble, when bailout is an option. (Keister 2016.) Another resent argument in favor of the bailouts, presented e.g. in Diamond and Rajan (2002) and Gorton and Huang (2004), is the liquidity effect. This means that banks insolvency reduces aggregate liquidity in the economy, which may escalate the problems further, and by bailing out the insolvent bank, government can add liquidity to the economy and prevent the problems arising from the liquidity shortage.

Contrary to arguments that support bailouts, there are also severe problems related to them. First, bailouts are expensive to taxpayers, and deteriorate the creditworthiness of the government. Second, the risk of the moral hazard is the most discussed disadvantage of bailouts. Banks have incentive to take excessive risks when they can rely on the government bailout if the risk realizes. (Bianchi 2016.) Also Diamond and Rajan (2002) argument that when bailout is not correctly targeted, it can lead to systemic crisis in the financial system. This is due to increase in excess demand for liquidity that leads to further insolvencies and bailouts.

The bailout cost and benefit analysis often concentrates on the different bailout policies or strategies instead of argumentation between bailout and no-bailout policies. E.g. Bianchi (2016) suggests that bailout has important stabilizing effects despite the moral hazard risks, when it is broad-based and not targeted to a single bank. Aghion et al. (1999) argue that bailouts should be strictly conditional, such that banks are obliged to liquidate the non-performing loans first to get the bailout. They also compare so called soft and hard bailout policies, where soft means less conditions for banks.

The effects of bailouts are usually analyzed with models containing several agents, most importantly depositors, banks, entrepreneurs and regulator, e.g. Aghion et al. (1999), Diamond and Rajan (2002) and Gorton and Huang (2004). The structure is usually such that entrepreneurs loan money from banks for projects, and the projects may fail causing entrepreneurs to default their debt. Banks need bailout when the average payoff from loans does not exceed the value of deposits issued to fund these loans. Sovereigns bailout decision, or the decision of bailout policy, is typically based on either maximizing expected social welfare, or minimizing the cost of excessive recapitalization, or to affect the banks management to evaluate their returns after the bailout.

In conclusion, one can state that some form of bailout policy is a plausible option for sovereigns now and in the future, and it transfers insolvency risk of banks to sovereign which is essential in formation of the feedback loop. Also, the academic research about bailouts can provide insight how the bailouts can be approached in the feedback loop studies.

2.4 Feedback loop

The feedback loop is often called "diabolic loop" in economic papers since it is a worsening self-perpetuating spiral. It connects the sovereign debt risk together with private banks' solvency. The basic theory about the feedback loop is quite simple. The loop can start from the unanticipated increase in sovereign debt risk or from solvency issues of a private bank. In the first case the deterioration of sovereign creditworthiness increases risk involved in the sovereign debt and thus increases the interest rates of government bonds. This in turn, reduces the market value of the government bonds that private banks hold as assets, and may lead to asset-liability mismatch in banks' balance sheet. In the worst case, bailout is required to save the bank. The other option is a so called credit crunch, meaning that the bank needs to reduce its lending. Both induce negative feedback to the sovereign, the bailout is of course a direct cost to sovereign, as the credit crunch decreases economic activity i.e. by decreasing investment, and leads to slower economic growth and losses in tax revenues. Based on these two effects Brunnermeier et al. (2016) divide the feedback loop in two parts, first is called "a bailout loop" and the second as "a real economy loop". The illustration of these feedback loop mechanisms is presented in Figure 3.

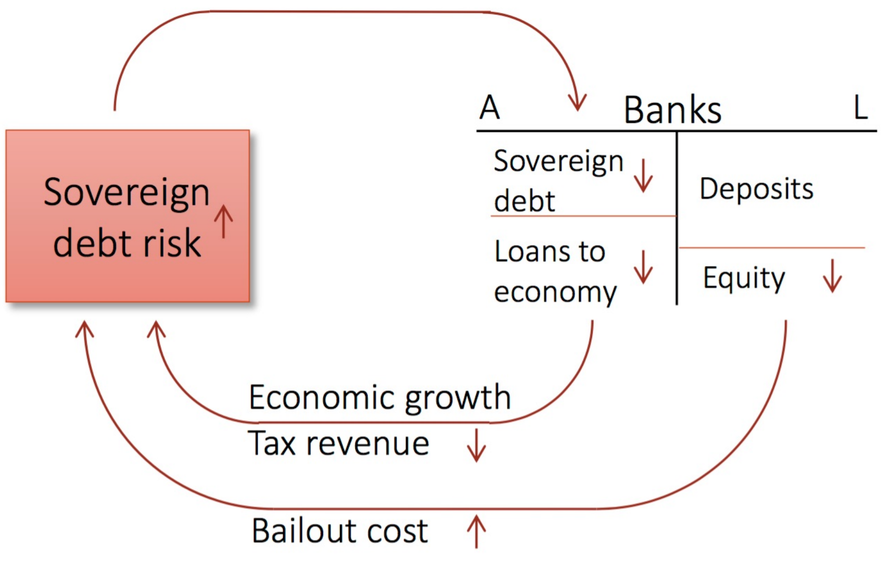

Figure 3: Two types of feedback loops, figure from Brunnermeier et al. (2016).

The second case is that feedback loop starts from the banking sector. The bank may become insolvent in example due to actualized credit risk, or because of a bank run. The insolvent bank needs to be bailed out. The bailout cost deteriorates sovereign credit worthiness inducing negative feedback to the banks' balance sheets and creating the pernicious loop. (Acharya et al. 2014.)

The feedback loop requires some basic ingredients, the most important of which are covered in more detail in previous chapters. First, home bias makes banks more exposed to sovereign debt risk than they would be with a more diversified portfolio (Battistini et al., 2014). Second, the bailout is an optimal choice for government in many situations and thus governments cannot effectively commit to no-bailout policies. Thirdly, free capital mobility, which enables international investors to allocate their investments to foreign government bonds whenever they think that the risk of domestic debt is increasing. Thus, investors' perception of the future sovereign solvency has direct effect on the price of the domestic debt. (Brunnermeier et al. 2016.)

Theory of the feedback loop became topical because of the Euro crisis, which was very persistent. One of the consequences of the crisis was that sovereign yields of the Eurozone countries started to diverge. This was illustrated in Figure 1 by showing the long-term government bond yields. Before 2008 the yields were almost the same in all Euro countries. After 2008 in the so called core countries of Eurozone, that Germany, France and Finland represent in figure 1, the yields remain on low levels and even start to decline, while in the so called peripheral countries, Greece, Portugal, Ireland, Spain and Italy, the yields increase and in the most severely affected countries, Greece, Portugal and Ireland, the yields rocket up very sharply. The divergence in yields is of course a result of the perceived risk involved in peripheral economies, and eventually is the core reason why economic research started pay attention to the possibility of the feedback loop. (Battistini et al. 2014.)

The origin of Eurozone crisis varies between countries. In Ireland, the starting point of the feedback loop was in the banking sector, where the financial crisis 2008 caused insolvency issues, which led to government bailout, and resulted in the increase in government default risk as illustrated in Figure 2. The reverse example is Greece, where public over-indebtedness became issue for private banks. In general, empirical studies, like Acharya et al. (2014), Battistini et al. (2014) and Bocola (2016), of the feedback loop concentrate solely on Euro crisis, and find strong empirical support for the feedback loop theory. As Farhi and Tirole (2016) state, there is exceptionally wide academic consensus about the feedback loop theory.

2.5 Banking union

The Euro crisis ignited a new development in European integration process, the creation of a banking union. The European banking union consists of three pillars: Single Supervisory Mechanism (SSM), Single Resolution Mechanism (SRM) and a single deposit insurance scheme (Béranger and Scialom, 2015). One of the initial reasons for the banking union was to prevent the feedback loop between banks and sovereigns (Constâncio, 2015).

There are several mechanisms in the European banking union aiming to cut the pernicious feedback loop. The SSM harmonizes banking supervision and rules across countries. Considering the feedback loop, the harmonization of the risk weights of banks' assets is essential. Also, the SSM simplifies and improves the supervision of large banks that operate in several countries. (Constâncio (2015).)

The key element in prevention of the feedback loop is the SRM. When the banking union is in action, the SRM is responsible for the resolution of distressed banks, including bailout decisions. The SRM contains a single resolution fund (SRF), which is funded by banks in the banking union and which is then used to cover the bailout costs (European Comission, 2015a). Before the SRF funds can be used to bailout banks, the Banking union rules require the use of so called bail-in, which means that banks shareholders and creditors must first cover banks liabilities at certain level. The bail-in is not only important to cover the banks losses without public funding, but it should also reduce excessive risk taking and moral hazard in banking business. (Constâncio (2015).)

The bail-in and the use of the SRF is limited to 8 % and 5 % of the total liabilities of a given bank respectively. Now, if this is still not enough to save the bank, the sovereign bailout can be considered. Based on the experiences from the Euro crisis, these thresholds should clearly be enough to prevent any government involvement. So, when the banking union is finished the situation leading to the feedback loop should be extremely rare, even if not entirely impossible. (Constâncio (2015).) However, the creation of the European banking union is a phased process, and especially the SRF funds are collected during a period of eight years (European Comission, 2015b). Like Béranger and Scialom (2015) argue, during the period when the banking union is under construction, the feedback loop is still possible, since the SRM may face situation in which it simply lacks enough resources to prevent the bank failure without the government involvement. Béranger and Scialom (2015), also pose a question about the final size of the resolution fund, which may run out in severe crisis, when many banks run into problems. In general it can be stated about the banking union, that the question of the feedback loop is not completely solved, but it transforms to a question about the credibility and the adequacy of the resources of the banking union.

3 Models of feedback loop in literature

This chapter presents a literature review of the theoretical feedback loop models. Since the beginning of Euro crisis, several papers have been published presenting models of the loop. Also, some empirical evidences supporting the theories have been studied. First, some general features and assumptions related to all the models are presented, and then closer look at the models and their results is reviewed.

3.1 General features of feedback loop models

Feedback loop models have been presented by several authors including Brunnermeier et al. (2016), Farhi and Tirole (2016), Cooper and Nikolov (2013) and Leonello (2014), while Acharya et al. (2014) and Bocola (2016) present a model and test it empirically.

The models of the feedback loop are rather complicated, since they all are multiple period models with multiple agents. A typical number of periods is 3, while Brunnermeier et al. (2016) use 4 periods, but these multi-period models are usually simplified, such that only a limited number of events occur each period, i.e. that consumption or investment decisions take place only on specific period. Bocola (2016) uses a slightly different approach, since he uses a neoclassical growth model enriched with a financial sector, hence the model does not have the same type of periodical structure than the other models. One common feature to all models is that they contain multiple agents. A sovereign or a government and banks are obvious agents in this context, but also these models often include consumers, other investors than the banks, and producers of consumption goods.

A general structure of the feedback loop models is following. On the first period government issues a bond with a certain face value and a price, and banks and other investors buy the bond with different shares. The government default risk follows an exogenous probability parameter (e.g. Brunnermeier et al. (2016)), or a state of the world drawn from an exogenous probability distribution (e.g. Farhi and Tirole (2016)). Based on the possible changes in the risk, the banks may face equity problems and require bailout during following periods, and this in turn affects the financial situation of the government in the last period of the model. The details differ substantially between papers, but this simplified structure describes the idea of the feedback loop modeling at a very general level.

Because the models are quite complicated, the authors make some simplifying assumptions to make the analysis more straightforward. Typical assumption present in some form in every paper presented here, is that there is no strategic behavior in sovereign default decision, meaning that default occurs only when sovereign cannot pay its debt. This seems plausible assumption, especially if Eurozone countries are considered. Other assumption present in many of the papers is that bailout is always beneficial to sovereign (e.g. Brunnermeier et al. (2016), Farhi and Tirole (2016)), or similar assumption that bailout does not occur only when sovereign does not have resources for it (e.g. Cooper and Nikolov (2013)). As presented in Chapter 2.3, in many cases the bailout is an optimal choice and thus this assumption is reasonable. It is worth mentioning that some authors do not do specifically modeling about the bailout channel, but handle government transfers to financial sector at a more general level.

3.2 Diabolic loop model

Brunnermeier et al. (2016) call feedback loop with descriptive name “diabolic loop”, and their model is referred here as diabolic loop model. They present a quite straightforward four period model, with four agents: government, depositors, banks’ equity holders and investors. They start by defining a one-country model and then extend it to a two-country model, where both countries are similar with each other. The key difference between these models is that in one-country model, investors and banks can only hold domestic bonds, while in two-country model foreign investment is also possible. Because the countries are symmetric, also foreign bonds contain risk and may be a source for banks’ insolvency, which differences in example from the model by Farhi and Tirole (2016), since they define foreign bonds as safe assets.

The basic idea in a diabolic loop model is, that exogenous probability parameter, the so called sunspot probability, defines whether the investors become pessimistic and thus reprice the domestic bond. Repricing may result in a bank becoming insolvent, and if that happens, the sovereign needs to decide if it bails out the bank. The sovereign can have either high or low surplus, again according to exogenously given probability, and only if the surplus is low the sovereign defaults its debt. If sunspot does not happen and no repricing occurs, sovereign never defaults. When defined like that, the model has two equilibria, the good one where no bailouts and no defaults ever happen, and the bad one, diabolic loop equilibrium where bailout happens and in the worst case leads to sovereign default. (Brunnermeier et al. 2016.)

The main event flow of the diabolic loop model is following. At period 0 government issues a bond, which investors and banks buy. Sunspot happens at period 1 and investors reprise the bond. At period 2 government decides whether to bail out banks. At the final period 3, government surplus is revealed and all consumption takes place. It is important to notice that government needs to decide about bailout before it knows the tax revenue, in other words government does not know if it can afford to bail out. If government does not bailout, the bank reduces lending and this in turn reduces tax revenue for the government, both reductions are presented as constant parameters. Brunnermeier et al. (2016) make several assumptions behind the diabolic loop model. The most important assumption that simplifies the model and its solution is, that the sovereigns fiscal surplus is always positive before bailout cost. The precondition of home bias in banks assets is assumed, making the one-country model applicable for analysis. Also, several parametric assumptions are done to focus the analysis primarily on the occurrence of the feedback loop. The diabolic loop model is more exactly presented in Chapter 4.

Using the diabolic loop model Brunnermeier et al. (2016) solve a threshold ratio of bank equity to sovereign exposure. When this threshold is exceeded, the feedback loop occurs. They also express the same result in a different way by presenting the maximum amount of sovereign debt that banks can hold safely, or as a minimum level of banks equity relative to sovereign debt it owns. The main purpose behind the analysis of Brunnermeier et al. is to study and present a solution to prevent the feedback loop from happening in Eurozone. Their suggestion is called European safe bonds, or ESBies, which is a joint bond of Eurozone countries that is split into a senior and a junior tranche. ESBies are rather complicated structural financial instruments, but the idea can be simplified such that the junior tranche contains risk, for investors willing to take more risk, and the senior trance is a safe asset. Thus, banks can be required to hold only the senior tranche of the ESBies. Using the diabolic loop model presented above, Brunnermeier et al. show that this tranche mechanism prevents the feedback loop, and they even come into conclusion that in equilibrium of the model also the junior tranche is a safe asset.

The diabolic loop model of Brunnermeier et al. (2016) is relatively straightforward and the paper aims to demonstrate the effectiveness of ESBies as a solution to Eurozone crisis. Nevertheless, the model is very intuitive in explaining the feedback loop itself and the conditions required to make it occur.

3.3 Macroprudential regulation

As in the previous chapter, also Farhi and Tirole (2016) use a representative name of the feedback loop by calling it “doom loop”. In their paper, they present a very elegant model of the feedback loop. The model has 3 periods and 4 types of agents: international investors, domestic bankers, domestic consumers and the sovereign. All the agents have utility functions which are linear over consumption. As usual, agents maximize their utility, and the sovereign optimizes welfare, which is defined as an expectation of the sum of agents’ individual utilities. Unlike Brunnermeier et al. (2016) who uses single probability to represent sunspot that affects investors, Farhi and Tirole uses a state of the world which follows an exogenous probability distribution, and this state determines the bankers’ balance sheets and the fiscal capacity of the sovereign. In mathematical sense, the maximization of expected utility in the model by Farhi and Tirole (2016) is done by integrating over the state of the world distribution. This makes the solution of the model bit more complicated looking than in Brunnermeier et al. (2016).

The simple presentation of the event flow of this model is pretty much like in Brunnermeier et al. (2016), except only three periods. On period 0 banks choose their portfolios by investing in domestic and foreign bonds. A state of the world is realized at period 1 and if a bank becomes insolvent and the sovereign decides to bail out, the sovereign issues new bond to cover the bailout costs. At period 3 the sovereign defaults, if it cannot afford to pay its debt with the current tax income. Again, like in Brunnermeier et al., the sovereign needs to decide whether to bailout before it knows its fiscal capacity. (Farhi and Tirole 2016.)

If the sovereign does not bail out, the banks reduce their lending which reduces investments. Now in the model investments are part of the welfare function, which the sovereign maximizes. On the other hand, the bailout cost is relative to the investment, such that the sovereign bailout is the portion of the investment that the bank cannot afford to. Now Farhi and Tirole assume that the return of investment is always greater than the cost of bailout, i.e. the bailout is always an optimal choice. (Farhi and Tirole 2016.)

One important difference between Brunnermeier et al. (2016) and Farhi and Tirole (2016) is that latter assumes foreign bonds as safe assets. Based on this assumption, Farhi and Tirole present a supervisory mechanism in the model. This means that exogenous supervisor can enforce banks to hold a certain level of safe assets, foreign bonds, and the role of the supervisory mechanism in the occurrence of feedback loops is one of the main subjects of the paper.

In the equilibrium of the model Farhi and Tirole (2016) solves the direct effect, that change in the state of the world has on the price of a sovereign bond. This solution shows that, if the probability of sovereign default increases, the price of a sovereign bond deceases, and the lower the value of a bond that a bank holds, the bigger the bailout it needs. Remembering that in this model, the sovereign funded bailout by issuing a new bond, the issuance of a new bond increases the supply of bonds and thus reduces the price of a bond even further.

So, this model precisely captures the mechanism of the feedback loop.

Regarding the role of the banking supervision, results are interesting. In the model's equilibrium, the consumption maximizing bankers always choose a portfolio where the amount of safe assets, foreign bonds, is on the lower bound enforced by the supervisor. This means that the bankers are rent seeking and trying to obtain the largest possible bailout. (Farhi and Tirole 2016.) Thus, this model also manages to capture the well-established moral hazard risk related to bank bailouts.

3.4 Role of bank runs

Cooper and Nikolov (2013) approach feedback loop modeling by combining two classical models: the Diamong-Dybvig bank run model (Diamond and Dybvig 1983) and Calvos pricing of government debt model (Calvo 1988). The basic structure of the model, including periods, agents and utility maximization is identical to above-presented Farhi and Tirole (2016) model. One of the interesting features of Coopers and Nikolovs model is that they analyze the feedback loop as a Nash equilibrium, where the players are the households, the banks and the government. Cooper and Nikolov show that the equilibrium is either the one where the government commits to no bailout policy and banks protect the deposits by issuing enough equity or the one where the government bails out. For the reasons discussed earlier in this paper, also Cooper and Nikolov state that the former, "good" equilibrium with no-bailout policy is not credible. Thus, the latter equilibrium prevails and the government bails out banks. The bailout means transfer from taxpayers to depositors, which makes depositors better off. As a result, rationally optimizing banks have little incentive to issue equity and they remain exposed to sovereign debt risk. Eventually, as a conclusion, the only reason for the feedback loop in this model is the moral hazard (Cooper and Nikolov 2013.)

Another slightly different model of the feedback loop is presented by Leonello (2014). The most notable peculiarity in her model is that the probability of crisis in both the banking sector and the sovereign is endogenous in the model, while all the other models have some form of exogenesis in the changes of the risk. The probability of both banking and sovereign debt crises is made endogenous using global games. Leonello also includes economic growth in the model and attaches the sovereign debt risk to the level of economic growth, meaning that the higher the growth the better the future solvency of the sovereign and the lower the risk of default. Similarly, with Cooper and Nikolov (2013) the bank runs are an essential feature in this model as well. Using this model Leonello shows that government guarantees are an essential channel that transfers bank credit risk to the sovereign. Whether the guarantees lead to a stable equilibrium or to an unstable feedback loop depends on the size and the scheme of guarantees. (Leonello 2014.) Leonellos model is somewhat limited compared with the others presented in here, since it does not include banks' holdings of government bonds, neither does it contain the real economy part of the loop, i.e. the lending reduction and its effect on tax revenue.

3.5 Models with empirics

A credit default swap (CDS) is a financial swap agreement that can be used to hedge the risk of a credit default. The seller of the CDS will compensate the buyer in the event of a credit default or other credit event. The price of the CDS increases if the risk of the underlying debt increases and vice versa. (Hull 2009 p. 518—522.) The CDS price changes reflect the changes in the investors' perception of the risk. Thus, the CDS price changes can be used to study the changes in a risk of, in example government bonds and bank credit. Acharya et al. (2014) and Bocola (2016) make us of the CDS data to find support to the mechanisms of the feedback loop.

Regarding the subject of this paper, the most interesting study is Acharya et al. (2014), since they first present a theoretical model, that is very similar with e.g. Farhi and Tirole (2016) and Brunnermeier et al. (2016). Then they present empirical evidence to support the main arguments of their model using CDS data from all Eurozone and five other European countries. They use two OLS regressions: first one to explain the prices of sovereign CDS's with banking sector risk, and the second to explain the changes of the prices of bank CDS's with sovereign risk. A multitude of control variables are used in regression models to rule out underlying unobserved factors that may affect both the sovereign and the bank credit risks. The authors also put lots of effort in testing the robustness of their results.

Empirical results of Acharya et al. (2014) provide support for two factors. First, the bank bailouts reduced banking sector credit risk, but increased sovereign risk simultaneously. Second, the sovereign credit risk increased the banking sector risk, but only after the crisis had started and first bailout happened. It is interesting to notice that before the Euro crisis, Acharya et al. find no correlation in either of their OLS-models. Also, the results show that the correlation from bank risk to sovereign risk is stronger for riskier countries. Acharya et al. (2014) is one of the initial papers to present a feedback loop as an explanation for Euro crisis and to present empirics supporting the theory of it.

Bocola (2016) presents different kind of approach from all the other models and papers presented in this chapter. He uses a neoclassical growth model enriched with a financial sector as a theoretical framework. The model is solved numerically, due to its complexity. Like Acharya et al. (2014), Bocola does the empirical analysis using CDS data, but only from Italy. Another peculiarity in this study is that the empirical analysis is done using Bayesian inference. Bocola focuses primarily on studying the effect of sovereign debt risk on a banking sector and further on a real economy. Thus, this study contributes to the feedback loop studies only partially. However, Bocola finds strong support for the transmission of sovereign debt risk to banks and from banks to further to real economy, and in that sense Bocola provides interesting insight about the real economy part of the feedback loop as well.

In this context, it is also suitable to mention Battistini et al. (2014), which uses CDS data from Eurozone to study home bias and the relationship between country and bank risks using multivariate SVAR-model. This study concentrates on some of the key elements of the feedback loop, but it is not exactly comparable to other studies presented here, since

Battistini et al. do not present any theoretical model of the feedback loop. Like two above-presented studies, also Battistini et al. finds evidence supporting the feedback loop theory.

4 One-country model

This chapter presents a one-country model which captures the logic behind the feedback loop. The model follows Brunnermeier et al. (2016). The overall idea behind the model is described in Chapter 3.2. This chapter first describes the general structure and some assumptions of the model and then proceeds to formalizing the model. All detailed calculations can be found from Mathematical appendix.

4.1 General structure and assumptions

There are four agents. First, the government, which prefers higher tax revenue over lower one. Second, domestic depositors, who pay taxes. Depositors withdraw their deposits if a bank becomes insolvent, unless the government bails out the bank. Third agent is banks equity holders, who invest all their capital as banks initial equity (E_0) and thus cannot re-capitalize banks in the future. This definition simplifies the modeling of government bailout. Fourth agent is investors, domestic and foreign, who invest in government bonds. The investors are the ones who determine the price of the government bond. For simplicity, all agents are assumed as risk neutral, and a risk free interest rate as zero. The model has four periods: $t = 0, 1, 2, 3$. All consumption, private and public, happens at $t = 3$.

There are two simplifying assumptions about the government fiscal surplus behind the model. First, governments primary surplus is assumed to be positive without a bailout cost. Second, if the governments surplus is high, the government can fully repay its debt even after a bailout. These assumptions are formalized later in this chapter.

4.2 Government surplus

Denote S to be the government primary surplus. With probability π the surplus is low $\underline{S}$ and with $1 - \pi$ high $\overline{S}$ $(> \underline{S})$. At $t = 0$, the government issues a bond, with face value $\underline{S}$, which means that initially government is not over-indebted, i.e. regardless of the surplus the government can fully repay its debt at $t = 0$. The market value of the bond is B_0. Now denote $\alpha \in [0, 1]$ as the share of the government debt the banks buy. Hence, at $t = 0$ banks hold αB_0 of the government debt as an asset in their balance sheet. The banks hold , also an amount L_0 of loans to the real economy. On the liability side of their balance sheets are deposits D_0 and equity E_0.

Denote p as the probability at which a sunspot occurs at period $t = 1$. This sunspot is purely random and does not contain any information about the state of the economy, e.g. current or future surplus of the government. Now, if a sunspot occurs, investors become pessimistic and reprice the government bond. Denote the new price with $B_1 (< B_0)$.

If repricing happens, banks suffer losses of $\alpha(B_1 - B_0)$. The banks end up with negative equity and thus become insolvent, if losses exceed the equity, i.e. $\alpha(B_1 - B_0) + E_0 < 0$. The insolvency decreases lending to $\psi \in [0, 1]$ share of the initial value, and the new lending is then ψL_0. Denote $\tau \in [0, 1]$ as a share of tax revenue which the government collects from the banks' lending. This can be interpreted in example as the taxes the government collects from investments funded with loans. Now, if banks become insolvent, government loses tax revenue on period $t = 3$ with the amount of $\tau\psi L_0$ (remembering that the realized surplus is revealed at final period $t = 3$). The term $\tau\psi L_0$ is also called the effect of a credit crunch, or simply a credit crunch.

The government must decide whether to bail out banks at period $t = 2$. If it does not bail out, another similar tax loss $\tau\psi L_0$ happens at period $t = 3$. The minimum size of the required bailout is the difference between losses and the initial equity, i.e. $\alpha(B_1 - B_0) + E_0$, which can be called as the endogenous cost of bailout.

Lets assume that a bailout happens. Then, the realization of sovereign surplus at $t = 3$, after the bailout, is

$$S - \tau\psi L_0 + \alpha(B_1 - B_0) + E_0 := S - C, \tag{1}$$

where S is a stochastic primary surplus at $t = 3$, without sunspot occurring, i.e. "surplus in a normal situation". On the right hand side of the equation 1, C denotes the total cost of bailout. This total cost is a sum of a credit crunch and an endogenous cost of bailout

$$C = \tau\psi L_0 - \alpha(B_1 - B_0) - E_0. \tag{2}$$

The bailout cost 2 is essential in the model and used in example in defining the price of a government bond in the next chapter.

4.3 Bond pricing

By definition, the government can always repay its debt, if the government surplus is high $\overline{S}$, even after bailing out banks. Now, if the surplus is low $\underline{S}$, the government can only pay $\underline{S} - C$ after a bailout, remembering that $\underline{S}$ was also the face value of the bond. This means that the market value of the bond at $t = 1$, is relative to the expectation of the surplus. Noticing that neither a bond repricing nor a government surplus realization happens at period $t = 2$, the bond value at $t = 1$ is the expected value of it at $t = 3$

$$B_1 = E(B_3) = (1 - \pi)\underline{S} + \pi(\underline{S} - C) = \underline{S} - \pi C. \tag{3}$$

Denote $\Delta_1 := \pi C$ to present the price discount of the bond. The value of the bond in the first period $t = 0$ is the expected value of the bond at $t = 1$. This value is relative to the probability of the sunspot. If the sunspot does not occur, repricing does not happen and the bond price is its face value, but if sunspot occurs, the price is the expected value after repricing, which is calculated in 3. Thus, the bond value at $t = 0$ is

$$B_0 = E(B_1) = (1 - p)\underline{S} + p(\underline{S} - \pi C) = \underline{S} - p\pi C. \tag{4}$$

Again, denote $\Delta_0 := p\pi C$ to present the price discount. Now, the banks suffer losses due to bond price decrease as

$$B_1 - B_0 = \underline{S} - \pi C - (\underline{S} - p\pi C) = -(1 - p)\pi C = -(1 - p)\Delta_1. \tag{5}$$

Combining the bailout cost equation 2 with equation 5, the price discount Δ_1 can be presented as

$$\Delta_1 = \pi C = \pi[\tau\psi L_0 - \alpha(B_1 - B_0) - E_0)] = \pi[\tau\psi L_0 - \alpha(-(1-p)\Delta_1) - E_0)], \quad (6)$$

and solved with respect to Δ_1

$$\Delta_1 = \frac{\pi(\tau\psi L_0 - E_0)}{1 - \pi\alpha(1-p)}. \quad (7)$$

Now, equation 7 presents the price discount without using the market prices B_0 or B_1. Next, this equation is used in defining the conditions for the feedback loop.

4.4 Feedback loop

The banks need bailout, if banks' equity is negative after bond repricing, i.e.

$$\alpha(B_1 - B_0) + E_0 < 0, \quad (8)$$

which, when plugging in the price loss equation 5, becomes

$$\alpha(-(1-p)\Delta_1) + E_0 < 0 \quad (9)$$

and further replacing Δ_1 with the exact price discount in 7 results

$$-\alpha(1-p)\frac{\pi(\tau\psi L_0 - E_0)}{1 - \pi\alpha(1-p)} + E_0 < 0. \quad (10)$$

The equation 10 simplifies to

$$\alpha(1-p)\pi\tau\psi L_0 > E_0 \quad (11)$$

Equation 11 presents an equity threshold of bailout. If inequality 11 holds, banks' equity is too low and they need bailout if sunspot occurs. On the other hand, the bailout is optimal for government only if the bailout cost is lower than the tax revenue loss from not bailing out banks. Presenting the bond price discount using equation 5 and further replacing Δ_1 with its exact form 7, a new form of discount can be presented as

$$B_1 - B_0 = -(1-p)\Delta_1 = -(1-p)\frac{\pi(\tau\psi L_0 - E_0)}{1 - \pi\alpha(1-p)}, \quad (12)$$

which can be slightly reorganized to

$$B_1 - B_0 = -\frac{(1-p)\pi}{1-\alpha(1-p)\pi}(\tau\psi L_0 - E_0) \tag{13}$$

Remembering that if the government does not bail out banks, a further $\tau\psi L_0$ credit crunch loss happens at period $t = 3$. Thus, the bailout is optimal choice if the bailout cost is lower than this credit crunch cost, i.e.

$$\alpha(B_1 - B_0) + E_0 > -\tau\psi L_0, \tag{14}$$

which, by plugging in 13 and reorganizing a little bit, becomes

$$E_0 > \alpha\frac{(1-p)\pi}{1-\alpha(1-p)\pi}(\tau\psi L_0 - E_0) - \tau\psi L_0. \tag{15}$$

Further simplifying equation 15 with respect to initial equity E_0 results to

$$E_0 > (2\alpha(1-p)\pi - 1)\tau\psi L_0. \tag{16}$$

Now, if banks' equity is below the threshold in 11 but above the bailout condition in 16, the sunspot leads to insolvency of the banks and they need a bailout and simultaneously the bailout is optimal for the government. This causes the feedback loop. Because the government primary surplus is low $\underline{S}$, the bailout cost forces the government defaulting part of the debt, since, like presented in chapter 4.3, the government can only pay $\underline{S} - C$ after a bailout. This default deteriorates governments creditworthiness and leads to new repricing of the bonds in the future resulting further losses to the banks, creating the feedback loop.

At this point, it is important to revisit and formalize some of the assumptions described only verbally at the beginning. First, at period $t = 1$ when sunspot occurs and the government suffers a tax loss, it is assumed that even if the government surplus is low $\underline{S}$, the tax loss is not making the government insolvent. Formally this is presented as $\underline{S} - \tau\psi L_0 > 0$. This assumption is necessary, because otherwise the government bailout would be completely unrealistic at $t = 2$, even if the bailout condition 16 holds. Also, part of the derivation of the model was done assuming in the background, that the banks equity is sufficiently small for the feedback loop. This means that, at least if banks hold all the government debt, i.e. $\alpha = 1$, the threshold condition 11 holds

$$\alpha(1-p)\pi\tau\psi L_0 > E_0 \Rightarrow (1-p)\pi\tau\psi L_0 > E_0. \tag{17}$$

This assumption was used in derivation just to make the whole scenario sensible. Of course, if the repricing of the bond would not make banks insolvent, then the increase in sovereign debt risk would be no problem what so ever.

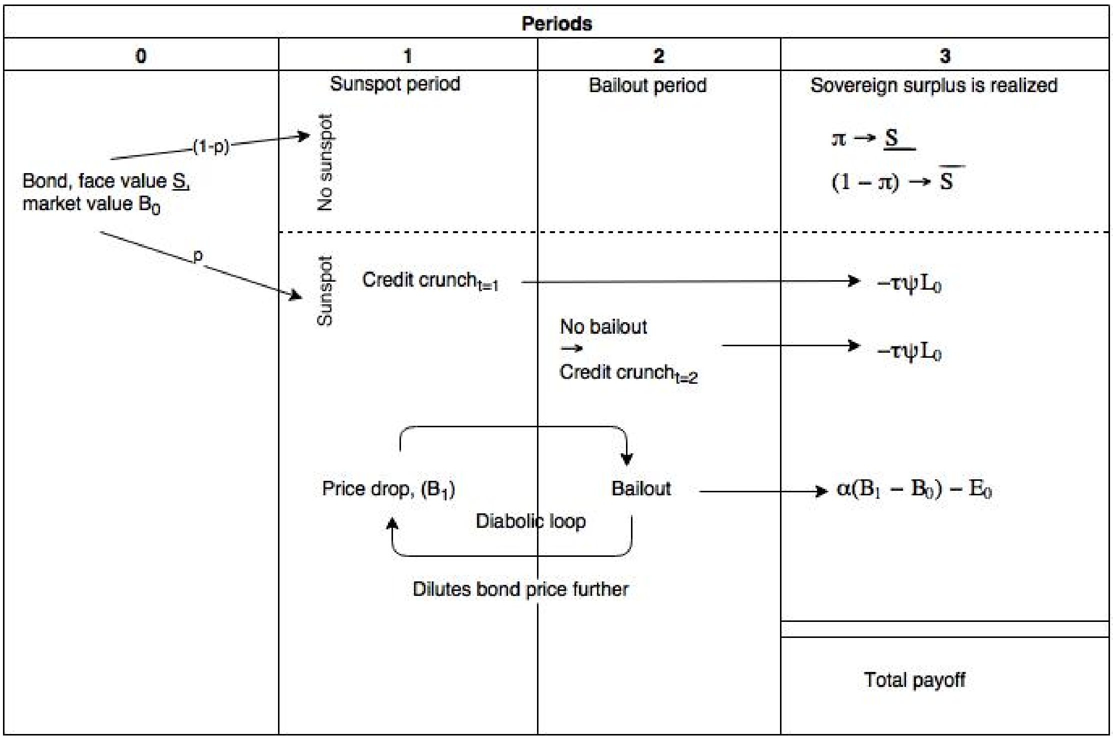

Figure 4: Time-line graph of the model.

A summary of the events in the model is illustrated as a time-line in figure 4.

4.5 Preventing feedback loop in one-country model

Now the conditions required for the feedback loop are known and the requirements that prevent the loop in the one-country model can be defined. This can be done by using the equity threshold for bailout in equation 11. If banks' equity is above this threshold, the bond repricing does not make banks insolvent, and thus the bailout is not needed when the sunspot occurs. The first implication is of course to set a minimum equity requirement for banks, which can be defined using the threshold equation 11. Denote $\underline{E_0}$ as equity lower bound, which can be defined as

$$\alpha(1-p)\pi\tau\psi L_0 := \underline{E_0} > E_0. \tag{18}$$

The banks must meet this $\underline{E_0}$ to avoid the feedback loop.

The feedback loop can occur only if the government has incentive to bailout banks. Thus, it is also worth checking what happens to the bailout condition 16, if banks' equity is $\underline{E_0}$. This can be done by comparing the bailout condition in 16 with the equity lower bound in 18

$$(2\alpha(1-p)\pi - 1)\tau\psi L_0 < E_0 < \underline{E_0} = \alpha(1-p)\pi\tau\psi L_0. \tag{19}$$

Remembering that $\alpha \in [0,1]$, $\pi \in [0,1]$ and $\tau\psi L_0 \geq 0$, the inequality in equation 19 simplifies to

$$\alpha(1-p)\pi < 1. \tag{20}$$

Inequality in 20 holds otherwise, except the borderline case of $\alpha = \pi = 1$ and $p = 0$. In this borderline case, the original inequality in 19 holds as equality, which means that the government would be indifferent between bailing out banks and not. Hence, one can conclude that if banks' equity is $\underline{E_0}$ no-bailout is never more optimal than bailout, and that the bailout condition holds. This further implies, that the feedback loop conditions are otherwise present, but the equity requirement $\underline{E_0}$ prevents it.

Now, instead of setting an equity requirement for banks, the feedback loop can be prevented by limiting banks' sovereign exposure. Using the minimum equity equation 18 the safe limit of banks' holdings of government bonds can be determined as

$$\alpha = \frac{\underline{E_0}}{(1-p)\pi\tau\psi L_0} \tag{21}$$

Now denote $\overline{\alpha}$ to present this safe limit of the bonds in 21. The maximum amount of sovereign debt available for banks is $\overline{\alpha}\underline{S}$. This can be presented more accurately by plugging $\underline{S}$ into 21, which results

$$\overline{\alpha}\underline{S} = \frac{\underline{E_0}}{(1-p)\pi\tau\psi L_0}\underline{S}, \tag{22}$$

which can be modified to

$$\frac{\underline{E_0}}{\overline{\alpha}\underline{S}} = \frac{(1-p)\pi\tau\psi L_0}{\underline{S}}. \tag{23}$$

This equation 23 present the minimum ratio of banks equity to sovereign exposure.

Replacing the face value of the government bond $\underline{S}$ on the left hand side of the equation 22 with the market value of the bond, results in equation that presents the maximum amount of safe assets available to banks

$$\overline{\alpha}B_0 = \frac{\underline{E_0}}{(1-p)\pi\tau\psi L_0}\underline{S}. \tag{24}$$

It is clear from all the equations 22 – 24, that the equity and sovereign exposure limits are relative to each other. So, in order to prevent the feedback loop either one needs to be controlled with respect to the other.

5 Two-country model

This chapter extends the one-country model to two-country version. The two-country model in a symmetric situation in which banks in two countries hold similar portfolios from Brunnermeier et al. (2016) is presented verbally at the beginning of this chapter and later also formalized using the formal two-country model. The formalization of two-country model follows the one-country model presented in Chapter 4 with some modifications. All detailed calculations can be found from Mathematical appendix.

5.1 Symmetric two-country case

Brunnermeier et al. (2016) describe the situation where the model contains another country and set these two-countries symmetric. The sunspot probability is independent between the two countries. Now, if banks hold only domestic bonds the situation does not change from the one-country model. In addition to the one-country case, there is a symmetric pool made of government bonds issued by the two countries. If banks diversify their portfolio to contain part of this pooled asset, banks are less exposed to domestic sovereign risk, and equity threshold is lower. This is the risk reduction benefit of diversifying.

However, if banks in both countries hold only this pooled asset, they end up with identical portfolios. If banks' equity is below the threshold, the repricing of either of the two bonds affects banks in both countries. This means that also the government in a country where sunspot did not occur needs to bail out its banks. Now identical portfolios with symmetric countries results that country risk is transferred to another country. (Brunnermeier et al. 2016.)

5.2 Formal two-country model

The following formalization is my own modification from the one-country model. The two-country model presented here also captures the results regarding the symmetric situation by Brunnermeier et al. (2016) described in previous chapter.

In the following formalization, notation follows the one-country model unless otherwise mentioned. Assume two similar countries. Both countries follow the model presented in chapter 4. The probability of sunspot for the first country is p, and probability of a low fiscal surplus $\underline{S}$ is π as earlier. The same probabilities for the second country are q and ρ respectively. These probabilities are independent from each other.

Banks are assumed to be free to choose their investment portfolio to consist of domestic bonds and foreign bonds. If banks hold only domestic bonds, the situation equals the one-country model. Now, denote P as portfolio that domestic banks choose of two bonds such that $P_0 = (\alpha B_0, \beta B_0^*)$, where α and β (α, $\beta \in [0, 1]$) are the shares of domestic and foreign sovereign debt the banks hold. Following the one-country model, B_0 and B_0^* are the market prices of domestic and foreign bonds at period $t = 0$ respectively, while the P_0 is the value of the whole portfolio at period $t = 0$. Here and onwards the superscript " $*$ " denotes foreign country variables.

First, two different portfolios can be compared. First portfolio contains only domestic bonds $P_0 = (\alpha_1 B_0)$, while the other one contains both domestic and foreign bonds $\hat{P}_0 = (\alpha_2 B_0, \beta B_0^*)$. If investment size is equal, i.e. $P_0 = \hat{P}_0$, the comparison results to

$$(\alpha_1 - \alpha_2) B_0 = \beta B_0^*. \tag{25}$$

Clearly, when $\beta > 0$ in equation 25 means that $\alpha_2 < \alpha_1$ (B_0 and B_0^* are assumed to be positive in this context). This shows the simple result that diversification reduces banks' exposure to domestic sovereign debt.

5.3 Cost of bailout and portfolio pricing

Similarly with equations 1 and 2 in the one-country model, the total cost of bailout, can be defined as the difference between banks' initial equity and the sum of credit crunch and endogenous bailout cost. The endogenous bailout cost is the loss from repricing the portfolio, which is calculated similarly than when repricing a single bond. So, the total cost is

$$C = \tau\psi L_0 - (P_1 - P_0) - E_0. \tag{26}$$

The portfolio pricing can be derived using similar backwards induction than in the bond pricing in the one-country model. The price at period $t = 1$ is the expectation of the value at final period $t = 3$, and the price at $t = 0$ is the expected price at $t = 1$. Individual bonds in the portfolio are priced according to equations 3 and 4. Now the prices for the whole portfolio can be determined as

$$P_1 = E(P_3) = \alpha E(B_3) + \beta E(B_3^*) = \alpha(\underline{S} - \pi C) + \beta(\underline{S}^* - \rho C^*) \tag{27}$$

and

$$P_0 = E(P_1) = \alpha E(B_1) + \beta E(B_1^*) = \alpha(\underline{S} - p\pi C) + \beta(\underline{S}^* - q\rho C^*) \tag{28}$$

Using these prices, the price discount of the whole portfolio is

$$\begin{aligned} P_1 - P_0 &= [\alpha(\underline{S} - \pi C) + \beta(\underline{S}^* - \rho C^*)] - [\alpha(\underline{S} - p\pi C) + \beta(\underline{S}^* - q\rho C^*)] \\ &= -\alpha(1-p)\pi C - \beta(1-q)\rho C^* \\ &:= -\alpha(1-p)\Delta_1 - \beta(1-q)\Delta_1^*, \end{aligned} \tag{29}$$

where $\Delta_1 = \pi C$ and $\Delta_1^* = \rho C^*$ denote the price discount of the domestic and foreign bond respectively.

5.4 Equity threshold and feedback loop

Previous chapter presented the price discount of the portfolio. Now, similarly with the one-country situation, the feedback loop occurs only, if banks are left with negative equity after portfolio repricing, i.e. they need a bailout. Using the bailout cost and the portfolio discount equations 26 and 29, the equity threshold can be solved. To make mathematical presentation a bit simpler, assume first that foreign bond discount Δ_1^* is unknown and the domestic bond discount Δ_1 is solved with respect to Δ_1^*,

$$\Delta_1 = \pi C = \pi[\tau\psi L_0 - (P_1 - P_0) - E_0], \tag{30}$$

which becomes, when the portfolio repricing is replaced with the exact form from equation 29,

$$\Delta_1 = \pi[\tau\psi L_0 + \alpha(1-p)\Delta_1 + \beta(1-q)\Delta_1^* - E_0], \tag{31}$$

and which by simply solving Δ_1 to the left hand side of the equation becomes

$$\Delta_1 = \frac{\pi[\tau\psi L_0 + \beta(1-q)\Delta_1^* - E_0]}{1 - \alpha\pi(1-p)}. \tag{32}$$

Now plugging 32 back into portfolio discount equation 29 results

$$P_1 - P_0 = -\alpha(1-p)\frac{\pi[\tau\psi L_0 + \beta(1-q)\Delta_1^* - E_0]}{1 - \alpha\pi(1-p)} - \beta(1-q)\Delta_1^*. \tag{33}$$

The banks end up with negative equity if portfolio price drop exceeds original equity.

$$P_1 - P_0 + E_0 < 0, \tag{34}$$

which can be presented using the exact portfolio discount equation 33,

$$-\alpha(1-p)\frac{\pi[\tau\psi L_0 + \beta(1-q)\Delta_1^* - E_0]}{1 - \alpha\pi(1-p)} - \beta(1-q)\Delta_1^* + E_0 < 0, \tag{35}$$

which simplifies, after some straightforward steps, to

$$\alpha(1-p)\pi\tau\psi L_0 + \beta(1-q)\Delta_1^* > E_0 := \underline{E_0}. \tag{36}$$

In equation 36 $\underline{E_0}$ denotes again the minimum equity required to avoid the feedback loop. So eventually, the equity threshold 36 that prevents the feedback loop is similar than in the one-country model, but this time the foreign bond discount is added. However, since there is the also foreign debt exposure, the sunspot in either of the two countries results to the feedback loop, if banks' aggregate equity is below the threshold in 36. If the sunspot occurs in the foreign country and the equity of domestic banks is below the threshold, some discount Δ_1^* occurs and the domestic government must bail out banks. This presents one form of the risk contagion that Brunnermeier et al. (2016) describe in a symmetric case.

5.5 Effect of foreign country

In previous chapter, the foreign bond discount Δ_1^* was assumed unknown. This discount depends on the structure of the foreign country, which can be specified more precisely to see how the foreign debt actually affects the domestic banks. Since both countries follow the same structure with different parameters, the foreign bond discount is similar with equation 32, but with foreign country's parameters, which are: probability of low fiscal surplus ρ, probability of sunspot q, share of domestic and foreign debt the banks hold α^* and β^* and banks initial lending and equity L_0^* and E_0^*. The exact form of discount is then

$$\Delta_1^* = \frac{\rho[\tau\psi L_0^* + \beta^*(1-p)\Delta_1 - E_0^*]}{1 - \alpha^*\rho(1-q)}. \tag{37}$$

To be exact with the notation, notice, that the share of domestic bond B_0 that domestic banks hold is α and the share of foreign banks' holdings is β^*. Similarly the shares of bond B_0^* are β and α^*.

First, assume that banks in foreign country are allowed to invest only to the foreign government bonds, i.e. their home country's bonds. Then the term $\beta^*(1-p)\Delta_1$ disappears from equation 37 and the equity threshold follows the one-country model (see equation 11 and chapter 4.4 for details). Now, if banks in the foreign country fulfill their equity threshold, by plugging the minimum equity equation 11 to 37 and using that in equation 36, the domestic banks' equity threshold simplifies to a sum of domestic and foreign equity thresholds weighted with the exposure parameters, i.e.

$$\underline{E_0} = \alpha(1-p)\pi\tau\psi L_0 + \beta(1-q)\rho\tau\psi L_0^*, \tag{38}$$

where $\tau\psi L_0^*$ is the size of a credit crunch in the foreign country. This situation is of course very special and limited, even somewhat unrealistic, but since the end result 38 is simple, it is useful in analysis and interpretation.

The general case where both countries are similar and banks are able to invest in the foreign and domestic bonds creates a cross dependence between countries. In this situation, it is assumed that both governments issue enough bonds such that banks in both countries can buy them. More precisely $\alpha + \beta^* \leq 1$ and $\beta + \alpha^* \leq 1$, remembering that the model contains also other investors besides banks, hence the inequality.

Now, the domestic bond discount, Δ_1 is 32 and foreign, Δ_1^* is 37 and the equity threshold is 36. Using these equations, the exact equity threshold can be solved, but the end results contains quite intricate coefficients. So, to keep equations more comprehensible, the following presentation contains additional notations. First, the familiar coefficient of domestic credit crunch is renamed: $\alpha(1-p)\pi := A \in [0,1]$ and with foreign weight parameter $\alpha^*(1-p)\pi := A^* \in [0,1]$. Second, two technical coefficients containing products of parameters $\alpha, \beta, \alpha^*, \beta^*, \pi, \rho, p$ and q are named as $\Theta \in]0,1[$ and $\Lambda \in [0,1[$ [3]. The exact derivation of the following equation is in Mathematical appendix. Using these notational modifications the equity lower bound in the general case, where banks in both countries hold bonds of both governments, becomes

$$\underline{E_0} = \frac{(\Theta - \Lambda)A}{\Theta}\tau\psi L_0 + \frac{\beta(1-q)\rho}{\Theta}[(1-A)(\tau\psi L_0^* - E_0^*) + \beta^*(1-p)\pi\tau\psi L_0]. \quad (39)$$

Notice that, in equation 39, $\Theta > 0$, which makes the equation valid with all the parameter values. The reason is that $\alpha\pi(1-p) < 1$ and $\alpha^*\rho(1-q) < 1$ in this context, because $\alpha < 1$ and $\alpha^* < 1$ since both of the bonds are held by the banks from both of the countries. Remembering that e.g. $\alpha = 1$ would mean that the domestic banks would hold the whole domestic sovereign debt.

Now, the equity threshold 39 is affected by foreign bond repricing, i.e. credit crunch $\tau\psi L_0^*$, but also the equity of the foreign banks E_0^*. No additional assumptions about this foreign equity is done, so it can be considered as an unknown variable, since solving also this equity would lead to excessively complex equations. The first part of sum in 39 is

[3] Precisely: $\Theta = (1 - \alpha\pi(1-p))(1 - \alpha^*\rho(1-q))$ and $\Lambda = \rho\beta^*(1-p)\pi\beta(1-q)$

interpreted as the effect of a domestic bond holdings and the latter part as the effect of foreign bond holdings. It is important to notice that the domestic bond repricing, i.e. the credit crunch $\tau\psi L_0$, is present in the second part of the sum as well. This means that part of the domestic bond repricing also affects now through the foreign bond holdings, since the foreign banks hold these same bonds as well.

The general form of equity threshold in equation 39 allows to study an another special case: the situation where the domestic banks only invest in the foreign bonds. This means that there is no direct exposure to the domestic sovereign debt. Technically this is done by setting $\alpha = 0$, which implies that $A = 0$ and $\Theta = 1 - \alpha^*\rho(1-q)$. Then equity threshold 39 becomes

$$\underline{E_0} = \frac{\beta(1-q)\rho}{1-\alpha^*\rho(1-q)}[(\tau\psi L_0^* - E_0^*) + \beta^*(1-p)\pi\tau\psi L_0]. \tag{40}$$

The first part of the sum in 39 disappears, since there is no more direct holdings of domestic bonds. However, in 40 there is still the effect of domestic credit crunch $\tau\psi L_0$ relative to probability of low domestic surplus π. The reason is that foreign banks hold bonds of both governments and the risk of both countries affects the risk of foreign banks' portfolio. Then the possible portfolio repricing affects the pricing of the foreign bond. Hence, the domestic sovereign risk reflects, through the foreign bond holdings, back to the domestic banks, even when the banks do not have direct exposure to the domestic sovereign debt.

6 Banking union

One of the recent developments in the European integration is the creation of a banking union. The reason for progressing towards the banking union in Eurozone is especially to prevent the feedback loop. The banking union consist of several different institutions, but in this context the interesting part is the Single Resolution Mechanism (SRM), which is responsible for the bailout decision if a bank fails. The bailouts are funded by a Single Resolution Fund, thus removing the cost from the member states of the union. The resolution fund is financed by banking sector. (European Comission 2015a.)

This chapter presents my own formal model of the banking union. The focus of the model is especially to study the effects that the banking union has in the bailout situations and its ability to prevent the feedback loop. The model covers the maximum cost of the banking union, the methods to collect funds from member states and different bailout scenarios. The model is based on the two-country model presented in previous chapter 5.

6.1 Cost of banking union

The banking union model is based on the two-country model presented in chapter 5. First, assume that the countries in the two-country model belong to a banking union, which is responsible for bailouts. In this context, the banking union is assumed not to have any other duties than to bailout insolvent banks. The first result is very straightforward. Assume that the sunspot still occurs as before and a repricing of banks' portfolios may result the banks to become insolvent. When governments are no longer responsible for the bailouts, the endogenous cost of bailout is removed from the cost equation 26, so the effect of credit crunch is the only cost that remains for the sovereign, thus

$$C = \tau \psi L_0. \tag{41}$$

But, already the one-country model was build with the assumption that even if the government surplus is low, $\underline{S}$, the tax loss is not making the government insolvent, i.e.

$\underline{S} - \tau\psi L_0 > 0$. This means that the banking union would break the bank to sovereign link in the model, if it covers to whole bailout cost.

Now, assume that a membership in the banking union is not free. Even though in SRM the resolution fund is financed by banks, some indirect cost can still be imposed to sovereigns. Assume that the banking union fee is relative to banks initial lending L_0. Denote $\omega \in [0, 1]$ as a share, which the banking union collects from banks to resolution fund, which finances the bailouts. This fee is collected at period $t = 0$, i.e. before the possible sunspot occurrence. The cost ωL_0 to banks further results in τ share decrease in tax revenue from lending. In the model, this mechanism corresponds the credit crunch in reprising situation, when banks cut their lending in an insolvency situation. This tax loss is the cost of banking union to sovereign. On the other hand, the lending decrease means that the initial lending is now only $(1 - \omega)L_0$ compared to original situation without the banking union cost. This further reduces the cost of credit crunch in a case of bailout. Now, the total cost of bailout to sovereign, including the banking union cost, is

$$C = \tau\psi(1 - \omega)L_0 + \tau\omega L_0 = \tau\psi L_0 + (1 - \psi)\tau\omega L_0, \tag{42}$$

in which the first term $\tau\psi(1 - \omega)L_0$ is the credit crunch from insolvency and second term $\tau\omega L_0$ is the cost of the banking union membership.

The membership in banking union is profitable to the sovereign if the cost in 42 is lower than the cost of bailout without banking union in the equation 26, e.g.

$$\tau\psi L_0 + (1 - \psi)\tau\omega L_0 < \tau\psi L_0 - (P_1 - P_0) - E_0, \tag{43}$$

which can be simplified to

$$-(1 - \psi)\tau\omega L_0 > P_1 - P_0 + E_0. \tag{44}$$

This inequality comparison requires of course a simplifying assumption that the banking union cost is collected only one time, and the bailout cost happens only one time. These assumption correspond the flow of events in one- and two-country models presented previously. Hence, the banking union model is consistent with the two-country model in this case.

The right hand side of the equation 44 is familiar from the equation 34, the inequality can be solved similarly than in equations 34–36. If the the left hand side of equation 44 is kept untouched, except the change of sign, the result is

$$(1-\psi)\tau\omega L_0 < \frac{\alpha(1-p)\pi\tau\psi L_0 + \beta(1-q)\Delta_1^* - E_0}{1-\alpha\pi(1-p)}. \tag{45}$$

On the right hand side of equation 45 are the cost of repricing the domestic bond, in which the parameter α is the share of domestic sovereign debt the banks hold, p the sunspot probability parameter and π the low fiscal surplus probability parameter, and foreign bond discount relative to share of foreign debt the banks hold β and the sunspot probability in foreign country q.

From equation 45, the upper bound to the banking union share of lending, ω, can be solved with respect to other parameters. The result, with slight reorganizing, is

$$\omega < \frac{\alpha(1-p)\pi\psi}{(1-\alpha\pi(1-p))(1-\psi)} + \frac{\beta(1-q)\Delta_1^* - E_0}{(1-\alpha\pi(1-p))(1-\psi)\tau L_0}. \tag{46}$$

This gives the maximum share of lending that the banking union membership can cost. ω in 46 is increasing in α and β, since bigger exposure to sovereign debt, domestic or foreign, increases the bailout cost, and decreasing in E_0, because the bigger equity means that the banks are able to cover lager share of the losses themselves. ω is also increasing in π, and noticing that Δ_1^* is presented in the equation 37, also increasing in ρ, meaning the larger the probability of low fiscal surplus, domestic or foreign, the higher the price of banking union can be. Interestingly ω is decreasing in τ and L_0. This is due to higher tax revenue, presented by τL_0. When the sovereign is able to collect more tax revenue from banks' lending the same share ω results in larger actual losses. Finally ω is decreasing in sunspot probability p and q, since the repricing risk is more priced into the higher sunspot probability bonds, and increasing in ψ, since the larger the credit crunch from insolvency the higher the bailout cost would be to a sovereign without the banking union.

Similarly the maximum share of lending to second country (foreign country) ω^* is determined with respect to foreign country parameters

$$\omega^* < \frac{\alpha^*(1-q)\rho\psi}{(1-\alpha^*\rho(1-q))(1-\psi)} + \frac{\beta^*(1-p)\Delta_1 - E_0^*}{(1-\alpha^*\rho(1-q))(1-\psi)\tau L_0^*}. \tag{47}$$

The endogenous cost of bailout, i.e. the difference between banks negative equity and portfolio repricing losses, is now moved from the sovereigns to banking union. Because the banking union is not affected by credit crunch, unlike the sovereign, the cost of bailout to the banking union is simply the banks' negative equity and nothing else. Condition for the banks' negative equity is presented in equation 34. By replacing he portfolio discount with the exact form from equation 33, the bailout cost of first country's banks to the banking union becomes

$$C = \alpha(1-p)\frac{\pi[\tau\psi L_0 + \beta(1-q)\Delta_1^* - E_0]}{1-\alpha\pi(1-p)} + \beta(1-q)\Delta_1^* - E_0, \tag{48}$$

which simplifies to

$$C = \frac{\alpha(1-p)\pi\tau\psi L_0 + \beta(1-q)\Delta_1^* - E_0}{1-\alpha\pi(1-p)}. \tag{49}$$

The cost of bailout of second country's banks is similar with foreign country parameters

$$C^* = \frac{\alpha^*(1-q)\rho\tau\psi L_0^* + \beta^*(1-p)\Delta_1 - E_0^*}{1-\alpha^*\rho(1-q)}. \tag{50}$$

6.2 Banking union funds

In previous chapter, the maximum cost of banking union membership to sovereign was presented in equation 45. This also gives a limit to funds that the banking union can collect. The banking union fee was ωL_0. The maximum fee can be solved from equation 45. This maximum fee is the limit that sovereigns are willing to pay for the membership, i.e. the membership is still profitable to them. Simply by multiplying L_0 from right hand side to left in equation 45, the higher bound for fee becomes

$$\omega L_0 < \frac{\alpha(1-p)\pi\psi L_0 + \beta(1-q)\Delta_1^* - E_0}{(1-\alpha\pi(1-p))(1-\psi)\tau}. \tag{51}$$

If equation 51 holds as equality the sovereign is indifferent in joining the banking union. Then, the right hand side of the equation 51 can be seen as the absolute maximum of the banking union fee. Denote this limit $\overline{\omega L_0}$ and similarly for the second country as $\overline{\omega^* L_0^*}$.

This gives the maximum total amount of funds that the banking union can acquire. Denote this amount of maximum funds as $\overline{F}$, which is then

$$\overline{F} = \overline{\omega L_0} + \overline{\omega^* L_0^*}, \tag{52}$$

and by replacing the maximum fees with the exact equations

$$\overline{F} = \frac{\alpha(1-p)\pi\psi L_0 + \beta(1-q)\Delta_1^* - E_0}{(1-\alpha\pi(1-p))(1-\psi)\tau} + \frac{\alpha^*(1-q)\rho\psi L_0^* + \beta^*(1-p)\Delta_1 - E_0^*}{(1-\alpha^*\rho(1-q))(1-\psi)\tau}. \tag{53}$$

The equation 53 is the borderline case of the banking union funds, which gives the maximum that is still feasible to member states. Since the banking union membership fee causes credit crunch that represses the economy in the member state, the banking union may not find it reasonable to charge the maximum fees presented in equations 52 and 53. One possible alternative is that the banking union fee is a fixed fraction of the maximum fee. Denote this fraction as μ and assume that μ is not country specific. Now, the banking union funds are

$$F = \mu\overline{F} = \mu\overline{\omega L_0} + \mu\overline{\omega^* L_0^*}, \tag{54}$$

and by replacing the maximum fees with the exact equations

$$F = \mu\frac{\alpha(1-p)\pi\psi L_0 + \beta(1-q)\Delta_1^* - E_0}{(1-\alpha\pi(1-p))(1-\psi)\tau} + \mu\frac{\alpha^*(1-q)\rho\psi L_0^* + \beta^*(1-p)\Delta_1 - E_0^*}{(1-\alpha^*\rho(1-q))(1-\psi)\tau}. \tag{55}$$

Second alternative is that the share of lending ω is not set country specifically, but the same share is used for both of the countries. Denote this general share as $\underline{\omega}$. Clearly, it must be selected such that $\underline{\omega} = min\{\omega, \omega^*\}$, because higher value would mean that joining banking union is not profitable to either one of the countries. In this case the banking union funds are

$$F = \underline{\omega} L_0 + \underline{\omega} L_0^*. \tag{56}$$

In example, if $\underline{\omega} = \omega$ the banking union funds can be presented as

$$F = \frac{\alpha(1-p)\pi\psi L_0 + \beta(1-q)\Delta_1^* - E_0}{(1-\alpha\pi(1-p))(1-\psi)\tau} + \hat{\mu}\frac{\alpha^*(1-q)\rho\psi L_0^* + \beta^*(1-p)\Delta_1 - E_0^*}{(1-\alpha^*\rho(1-q))(1-\psi)\tau}. \tag{57}$$

In equation 57 the $\hat{\mu}$ is not exogenously chosen like μ in 55, but presents the difference between $\omega^* L_0^*$ and $\underline{\omega} L_0^*$ such that

$$\hat{\mu}\omega^* L_0^* = \underline{\omega} L_0^*, \tag{58}$$

from which the $\hat{\mu}$ can be solved as

$$\hat{\mu} = \frac{\underline{\omega}}{\omega^*}. \tag{59}$$

Equations 58 and 59 show that $\hat{\mu}$ can be interpreted as the fraction of the maximum banking union fee for the second country. In that sense, it is congruent with μ in equations 54 and 55.

6.3 Bailout scenarios in banking union

This chapter presents the different bailout scenarios when the banking union funds are collected using those methods presented in previous chapter 6.2. Because the banking union is a supranational institution, it is inaccurate to refer to two countries in the model as domestic and foreign as is done previously when presenting the two-country model. That is why in this chapter they are called simply as country one and two, where country one corresponds the domestic and two foreign in previous presentation. In two country setup there is three possible bailout situations for the banking union, which are: banks in country one need to be bailed out, banks in country two need to be bailed out or banks in both countries need to be bailed out simultaneously. Of course, the first two scenarios are identical because both countries follow similar model with only parametric differences. Also, there is the fourth situation that no bailout is needed at all, but that is uninteresting in this context.

To be successful in bailing out banks, the banking union needs to posses enough funds to finance the bailout. This means that the total cost of bailout C^{TOT} is lower than the banking union funds, i.e.

$$C^{TOT} \leq F, \tag{60}$$

which in worst-case scenario is

$$C^{TOT} = C + C^* \leq F, \tag{61}$$

meaning banks in both countries need a bailout simultaneously.

The first alternative is that banking union fee is the maximum that is still affordable to member states. Then, the amount of funds in the banking union is the absolute maximum that the union can collect, $\overline{F}$. Now, the comparison of bailout cost and banking union funds, using equations 61 and 52 becomes

$$C^{TOT} = C + C^* \leq \overline{F} \Leftrightarrow C + C^* \leq \overline{\omega L_0} + \overline{\omega^* L_0^*}. \tag{62}$$

In this case the comparison of costs and funds can be simply done by comparing first the cost of bailout in country one with fees payed by banks from country one, e.g. taking only first part of 62

$$C \leq \overline{\omega L_0}. \tag{63}$$

This can be done more precisely by taking the exact forms from equations 49 and 53 which results

$$\begin{gathered} \frac{\alpha(1-p)\pi\tau\psi L_0 + \beta(1-q)\Delta_1^* - E_0}{1 - \alpha\pi(1-p)} \\ \leq \\ \frac{\alpha(1-p)\pi\psi L_0 + \beta(1-q)\Delta_1^* - E_0}{(1-\alpha\pi(1-p))(1-\psi)\tau}. \end{gathered} \tag{64}$$

Noticing that $(1-\psi)\tau \in [0, 1]$ by definition, the inequality in 64 always holds. Of course the same result applies to country two, since the two countries in the model are similar. This means that when the banking union collects the maximum fees, the funds acquired from a country always exceeds the cost of bailout of the banks in that country. There are two reasons for this result. First, the banking union does not suffer from credit crunch like sovereign does in bailout situation, thus the term $(1-\psi)$ on the right hand side of equation 64. The second reason is that the banking union fee only affects the sovereign relative to the share of taxes lost due to the banking union membership fee, which is presented by the term τ. Hence, the maximum funds are enough to cover bailout costs, even if banks in both countries need the bailout simultaneously.

The second alternative is that the banking union fee is only a fixed fraction of the maximum. This situation is presented in equations 54 and 55. First, the scenario where banks in country one need a bailout can be presented similarly than the previous scenario in 63, i.e.

$$C \leq \mu\omega L_0. \tag{65}$$

Following the equation 64, the exact form of cost is 49, which is this time compared to funds equation 55

$$\begin{gathered}\frac{\alpha(1-p)\pi\tau\psi L_0 + \beta(1-q)\Delta_1^* - E_0}{1-\alpha\pi(1-p)} \\ \leq \\ \mu\frac{\alpha(1-p)\pi\psi L_0 + \beta(1-q)\Delta_1^* - E_0}{(1-\alpha\pi(1-p))(1-\psi)\tau}.\end{gathered} \tag{66}$$

Clearly if $\mu \geq (1-\psi)\tau$, inequality 66 holds. Again, the same result applies to comparison with country two. So, the situation does not change from previous maximum funds case and the banking union is always successful in bailouts.

Now, if $\mu < (1-\psi)\tau$ in 66 the fees paid from country one are not enough for bailout costs of that country's banks. Noticing that all of these parameters ψ (decrease in lending due insolvency), τ (share of taxes collected from lending) and μ (share of the maximum banking union fee) are common for both countries, the same result applies to country two as well. The immediate implication is that in this case the banking union cannot afford to bail out banks from both countries simultaneously. However, the bailout of banks in country one is successful if funds from both countries are larger than the cost of bailout of banks from country one, i.e. if

$$\begin{gathered}\frac{\alpha(1-p)\pi\tau\psi L_0 + \beta(1-q)\Delta_1^* - E_0}{1-\alpha\pi(1-p)} \\ \leq \\ \mu\frac{\alpha(1-p)\pi\psi L_0 + \beta(1-q)\Delta_1^* - E_0}{(1-\alpha\pi(1-p))(1-\psi)\tau} \\ + \\ \mu\frac{\alpha^*(1-q)\rho\psi L_0^* + \beta^*(1-p)\Delta_1 - E_0^*}{(1-\alpha^*\rho(1-q))(1-\psi)\tau}.\end{gathered} \tag{67}$$

This equation 67 simplifies to

$$
\begin{gathered}
(1-\frac{\mu}{(1-\psi)\tau})\frac{\alpha(1-p)\pi\tau\psi L_0+\beta(1-q)\Delta_1^*-E_0}{1-\alpha\pi(1-p)} \\
\leq \\
\mu\frac{\alpha^*(1-q)\rho\psi L_0^*+\beta^*(1-p)\Delta_1-E_0^*}{(1-\alpha^*\rho(1-q))(1-\psi)\tau}.
\end{gathered}
\tag{68}
$$

Now, on the left hand side of 68 is a coefficient from exogenous parameters and the bailout cost bailout of country one, while the right hand side is funds gathered from the country two. In simple terms this can be expressed as

$$
(1-\frac{\mu}{(1-\psi)\tau})C \leq \mu\omega^* L_0^*. \tag{69}
$$

Equation 69 states that, after all the funds collected from country one are used, the remainder of the bailout cost must be less or equal than the funds collected from country two. One of the key factors that define whether 69 holds is the amount of initial lending in the two countries L_0 and L_0^*, which can also be interpreted as a proxy for the size of a banking sector in that country. So, in this case, if L_0^* is sufficiently small, it is possible that the banking union fails to bailout the banks in country one entirely, and on the contrary the banking union can afford to bail out banks in country two, due to smaller banking sector and thus smaller bailout cost.

The third alternative to collect funds is that banking union uses the same share ω for both countries. Assume that in this case the smaller share is ω and thus chosen as the general share, formally $\underline{\omega}=\omega$. Then, the banking union funds can be presented as in equations 56. If banks in country one need bailout, the banking union can afford to do it if

$$
C \leq \underline{\omega}L_0+\underline{\omega}L_0^* = \omega L_0+\hat{\mu}\omega^* L_0^*. \tag{70}
$$

Replacing the bailout cost and the funds collected from country one with the exact equations from 49 and 57, while keeping the equation for funds collected from country two as they are in 70, the comparison becomes

$$\begin{gathered}\frac{\alpha(1-p)\pi\tau\psi L_0+\beta(1-q)\Delta_1^*-E_0}{1-\alpha\pi(1-p)}\\ \leq \\ \frac{\alpha(1-p)\pi\psi L_0+\beta(1-q)\Delta_1^*-E_0}{(1-\alpha\pi(1-p))(1-\psi)\tau}\\ + \\ \hat{\mu}\omega^* L_0^*.\end{gathered} \tag{71}$$

It is directly seen from 71 that situation of country one corresponds to maximum funds case. This means that inequality 71 always holds for country one, i.e. the banking union is always able to bail out banks in country one. The more interesting situation is then the one, in which the bailout is needed for banks in country two. Formally the comparison is then

$$C^* \leq \omega L_0 + \hat{\mu}\omega^* L_0^* \tag{72}$$

Now, replacing the cost and funds with exact forms from the equations 50 and 57, the comparison between cost and funds becomes

$$\begin{gathered}\frac{\alpha^*(1-q)\rho\tau\psi L_0^*+\beta^*(1-p)\Delta_1-E_0^*}{1-\alpha^*\rho(1-q)}\\ \leq \\ \frac{\alpha(1-p)\pi\psi L_0+\beta(1-q)\Delta_1^*-E_0}{(1-\alpha\pi(1-p))(1-\psi)\tau}\\ + \\ \hat{\mu}\frac{\alpha^*(1-q)\rho\psi L_0^*+\beta^*(1-p)\Delta_1-E_0^*}{(1-\alpha^*\rho(1-q))(1-\psi)\tau}.\end{gathered} \tag{73}$$

In this case, the situation corresponds to previous scenario, in which only a fraction of the maximum, μ, was used when defining the banking union membership fee. So, similarly with that scenario, also here if $\hat{\mu} \geq (1-\psi)\tau$, the inequality 73 always holds, since funds collected from country two exceed the cost of bailout of banks in country two. In the opposite situation $\hat{\mu} < (1-\psi)\tau$, the solution can be done similarly with equations 67 –

69 to get the simplified equation

$$(1 - \frac{\hat{\mu}}{(1-\psi)\tau})C^* \leq \omega L_0. \tag{74}$$

Interpretation is also similar than earlier. If the reminder of the cost, after using all the funds collected from country two is less or equal than funds collected from country one, the bailout is successful.

The case of bailout in country two extends to final situation where both countries need bailout simultaneously. The banking union possesses enough funds to do this worst-case bailout if

$$C + C^* \leq \omega L_0 + \hat{\mu}\omega^* L_0^*, \tag{75}$$

which is in the exact forms, from equations 49, 50 and 57

$$\begin{gathered}\frac{\alpha(1-p)\pi\tau\psi L_0 + \beta(1-q)\Delta_1^* - E_0}{1-\alpha\pi(1-p)} \\ + \\ \frac{\alpha^*(1-q)\rho\tau\psi L_0^* + \beta^*(1-p)\Delta_1 - E_0^*}{1-\alpha^*\rho(1-q)} \\ \leq \\ \frac{\alpha(1-p)\pi\psi L_0 + \beta(1-q)\Delta_1^* - E_0}{(1-\alpha\pi(1-p))(1-\psi)\tau} \\ + \\ \hat{\mu}\frac{\alpha^*(1-q)\rho\psi L_0^* + \beta^*(1-p)\Delta_1 - E_0^*}{(1-\alpha^*\rho(1-q))(1-\psi)\tau}.\end{gathered} \tag{76}$$

Equation 76 can be simplified by grouping terms similarly than is done e.g. in equations 67 – 69. After moving all terms to the left hand side of the equation and replacing cost equations with their simple notations, the result is

$$(1 - \frac{1}{(1-\psi)\tau})C + (1 - \frac{\hat{\mu}}{(1-\psi)\tau})C^* \leq 0. \tag{77}$$

In equation 77, the first term is certainly negative or zero, since $(1-\psi)\tau \leq 1$. The sign of the second term is ambiguous. If $\hat{\mu} < (1-\psi)\tau$, the second term is positive. If the first term is at the same time strictly negative, the overall result depends on the sizes of the first and second terms. There is no straightforward interpretation of this case anymore, because the sign of the final summation in 77 depends on many variables within the costs C and C^*. In general, the major factors are the relative size of banking sectors in two countries presented by L_0 and L_0^*, but also the solvency of the banking sector within a country e.g. the size of L_0 compared to E_0. How different parameters affect the banking union membership fee and thus the amount of funds it has, are presented in previous chapters 6.1 and 6.2, while the factors affecting the cost of bailout are explained in detail as part of the two-country model in chapter 5.3.

Finally, consider a situation in latter two cases where the banking union does not have enough funds to bail out banks in both countries. For simplicity, assume that the cost of bailout in country one equals the whole banking union funds, i.e. $C = F$, and that the worst case happens and the banks in country two need bailout as well. Assume also that the banking union fee is μ fraction of the maximum fee denoted here as $\overline{\omega}$. Now, the banking union bails out the banks in country one, but the government in country two must decide whether it bails out the banks in country two or not. In this case the bailout cost to sovereign becomes

$$C^* = \tau\psi(1 - \mu\overline{\omega^*})L_0^* - (P_1^* - P_0^*) - E_0^* + \mu\tau\overline{\omega^*}L_0^*, \tag{78}$$

in which the total cost consists of credit crunch from banks' insolvency (the first term in 78), a bailout cost and the credit crunch from the banking union membership fee (last term in 78). This cost can be compared to the bailout cost without the banking union membership, which is presented in equation 26, e.g. in equality

$$\tau\psi L_0^* - (P_1^* - P_0^*) - E_0^* = \tau\psi(1 - \mu\overline{\omega^*})L_0^* - (P_1^* - P_0^*) - E_0^* + \mu\tau\overline{\omega^*}L_0^* \tag{79}$$

Equation 79 simplifies with couple of straightforward calculation steps (calculation details in Mathematical appendix) into a simple equation

$$0 = (1 - \psi)\mu\overline{\omega^*}. \tag{80}$$

The equality in 80 holds only if $\psi = 1$ or $\mu\omega = 0$. The former would mean, that banks insolvency would cut out the whole lending of the bank and the latter, that the banking union membership is free. So either way the equality is very extreme borderline case. Notice also that the right hand side of 80 is by definition always non-negative. This means that also the right hand side of equation 79 is always larger or equal than the left hand side. This in turn means that the bailout cost to the sovereign is larger or equally large as a member of the banking union than outside of it. In the case when the banking union fails to cover any part of the bailout cost of country two, the banking union membership is most likely disadvantageous for country two.

6.4 Consistency of banking union model

The banking union model presented here is based on the two-country model in chapter 5. Hence, the banking union model needs to be consistent with the two-country model. In this regard, the use of banking union funds needs further examination. Consider situation in which the sunspot occurs and the banking union does a bailout using all of its funds. Now, if the banking union funds cover exactly the whole bailout cost, the sovereign does not need to do anything and the banks lose the funds collected by the banking union. This corresponds the situation in the two-country model, in which the sovereign surplus is high, $\overline{S}$, and the sovereign always remains solvent. The same applies, if the banking union does not have enough funds and the sovereign needs to pay part of the bailout, but it remains solvent. In the worst-case scenario, the sovereign pays part of the bailout and its solvency deteriorates, then the feedback loop occurs. All of these situations are consistent between the banking union model and the two-country model.

The second alternative is that the sunspots do not occur in neither of the countries, or the banking union bails out banks but has more funds than the bailout costs are. This leaves all or some of the banking union funds unused. Without additional assumptions, this would mean that this banking union surplus simply disappears at period $t = 3$. To be consistent with the two-country model, assume that the banking union returns all surplus back to the banks at period $t = 3$ in correct shares between countries. Now, if sunspots do not occur, all of the membership fees are returned to the banks, and situation corresponds to a stochastic primary surplus realization in the two-country model, i.e. "surplus in a normal situation". In latter case in which some funds are used to bailout banks, the situation corresponds to the high sovereign surplus situation, in which the sovereign always remains solvent.

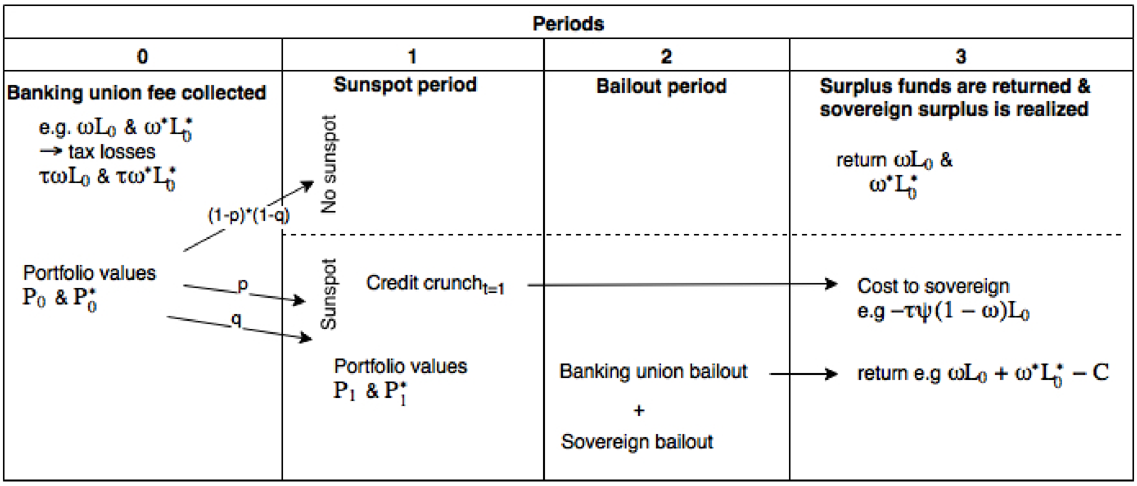

Figure 5: Time-line of the banking union model.

The time-line of essential events in the banking union model, including the above made assumption of returning surplus funds to banks, is presented in Figure 5. At $t = 0$ the banking union fee is collected from banks, which results a tax loss to sovereign. For the sovereign, this tax loss is permanent, even though the whole fee may be returned to the banks at period $t = 3$. Hence, the tax loss does not depend on the sunspot probability. For the banks, the occurrence of sunspot results to losses. If these losses are larger than the banks' initial equity, the banks need a bailout. The banking union performs a bailout at period $t = 2$. If the banking union does not possess enough funds to pay the whole bailout, the sovereigns may decide to complete the bailout. Now, this sovereign bailout happens at the same period $t = 2$ than the banking unions bailout. This is consistent with the assumption that all information is available to all agents in the model, hence the sovereign is able to do the bailout decision parallel with the banking union. By defining that the unused funds are returned to banks, the banking union model is consistent with the one- and two-country models, on which it is built.

7 Analysis

This chapter contains analysis based on the one- and two-country models presented in Chapters 4 and 5. Both models show that banks' aggregate equity and the exposure to sovereign debt are key factors in the occurrence, but especially in the prevention of the feedback loop. The two-country model also shows that risk may transfer from country to another. The banking union cuts the bailout channel, and thus prevents the feedback loop in the model.

7.1 Feedback loop in one-country model

The conditions when the feedback loop occurs in the one-country model are solved in Chapter 4.4. The essential factor is banks' aggregate equity. If the equity is low, below the threshold in equation 11, but sufficiently large such that a bailout is an optimal choice, i.e. above the bailout limit in 16, the sovereign bond repricing leads to a feedback loop.

The second important factor for the feedback loop is the situation of sovereign's economy. In the model, this is presented by the probability of low fiscal surplus π. Low surplus causes a budget deficit to sovereign, if it faces additional costs, a bailout cost and a tax reduction due to a credit crunch. Hence, the equity threshold 11 is increasing in π. The bond price difference equation 13 shows that the price drop is bigger the bigger the π is. The low surplus leads to a bigger price drop and thus more equity is required from banks.

Thirdly, the equity threshold is increasing in sovereign debt exposure presented by parameter α. This is a rather obvious outcome, since the larger is the share of the sovereign debt, the larger is the effect of the repricing of that debt. Since banks can choose the amount of bonds they buy, i.e. decide the exposure level α, and they can affect the amount of equity they possess, the solution to feedback loop in equation 23 is solved and presented as a minimum ratio of banks' exposure to equity.

Finally, the last factor affecting the feedback loop is the risk level of sovereign debt, which is presented with the probability of sunspot p. The higher sunspot probability means that investors are more likely to become pessimistic about the sovereign's economic development. This can be interpreted as a higher overall risk level of sovereign debt. Now in the model, all the probabilities are known by all the agents, thus when the bond prices are calculated in Chapter 4.3 the sunspot probability is taken into account. This means that higher sunspot probability, higher risk, leads to a lower bond price. The lower price leads to a smaller price drop when sunspot occurs e.g. in 13, and that is why the equity threshold is decreasing in sunspot probability p. In general, this is an interesting result, because it means also that safer bonds require more equity from banks, since very unlikely sunspot is less priced.

The one-country model also shows that bailout is the optimal choice, if the cost from bailout is less that the cost of no-bailout decision. This bailout condition is presented as equation 16 and it behaves similarly than the equity threshold with respect to different parameters. So, the lower limit of profitable bailout increases when the exposure to sovereign debt or the probability of low government surplus increases, and decreases when sunspot probability increases. Technically, the cost of no bailout is easy to implement. Here, it is straightforwardly just an another credit crunch related tax revenue decrease. In practice however, the cost of bank failure may be impossible to quantify, and thus the bailout condition may be considered more as a technical curiosity than an important outcome of the research.

A conclusion from the one-country model is that the feedback loop can be avoided by requiring banks to hold sufficient amount of equity $\underline{E_0}$ given the banking systems exposure to sovereign debt α. This creates a limit to the supply of safe assets, i.e. feedback loop free amount of sovereign debt, which is $\overline{\alpha}\underline{S}$ derived in Chapter 4.5. Brunnermeier et al. (2016) notice that this debt exposure limit also effectively limits the amount of safe deposits that banking system can generate. Hence, in practice this kind of equity requirement may turn out to be too strict and limiting as a policy recommendation.

7.2 Feedback loop in two-country model

The basic results of the reasons behind the feedback loop in the two-country model follow the ones derived with one country. Again, the fundamental issue is the amount of equity in banks' balance sheets. If the equity is too low, the repricing of banks' investments leads to a feedback loop like in the one-country situation. The more important results from the two-country model are related to interaction between the credit risk related to these countries and their banks.

The big difference between one- and two-country models is that in the two-country version banks have the possibility to diversify their investments by holding foreign bonds beside the domestic ones. Thus, the home bias is reduced. The first result, that reallocating portfolio from one type of bond to two different types reduces the direct exposure to single bonds risk, is rather obvious. This can be seen by a simple comparison of portfolios, like in equation 25, or from equity thresholds like 36 and 39, where reallocating a portfolio of the same size means decreasing α and increasing β.

More formally, comparing a portfolio that contains only domestic bonds, with a portfolio where part of the domestic bond holdings is reallocated to foreign bond holdings can be demonstrated by changing the exposure parameter values e.g. in the first portfolio $\alpha_1 > 0$ and $\beta_1 = 0$ and in the second portfolio $\alpha_2 < \alpha_1$ and $\beta_2 = \alpha_1 - \alpha_2$. If all other parameters remain unchanged, the domestic bond price drop affects the price drop of the whole portfolio in equation 33 less with α_2 than with α_1.

Now from the feedback loop point of view the situation changes from the one-country model such that the equity threshold depends also on the foreign debt repricing. In the simplest case, where foreign banks hold only their own government bonds and meet the minimum equity requirement is shown in equation 36. This special case is very straightforward and the equity threshold is simply a sum of domestic and foreign country's thresholds from the one-country model weighted with exposure parameters α and β. Now, if banks' aggregate equity is below this threshold, sunspot in either of two countries results in feedback loop. Somewhat more complicated looking equation 39 in the general case where banks in both countries hold the bonds of both governments verifies this result.

A symmetric case by Brunnermeier et al. (2016), where banks hold similar portfolios, means that the equity threshold for the banks of both countries follows equation 39. The repricing effect of bonds is represented by $\tau\psi L_0$ and $\tau\psi L_0^*$, and in the symmetric case these are similarly present in both countries. So, the repricing of the portfolio cannot happen in one country without happening in the other. Thus, if equity is below the threshold, either one of the two repricing effects results in the feedback loop. And, if banks' equity is too low in both countries the feedback loop occurs in both countries, even when the repricing happens only to one of the bonds in the portfolio. This phenomenon Brunnermeier et al. call the perfect contagion.

The feedback loop can be prevented like in one country situation by requiring banks to hold enough equity. However, the equity threshold is now determined by the foreign country specific parameters as well. Even in the simplest case 36, the threshold equation includes the lending of the foreign banks L_0^* which affects the foreign credit crunch and tax revenue. More complicated cases like in equation 39, when no additional assumptions are made about the foreign banks' initial equity, this foreign equity affects the equity requirement of domestic banks. Also, the portfolio allocation decisions made by foreign banks, represented by α^* and β^* are present in equation 39. This means that the minimum equity required to prevent the feedback loop also depends on the portfolio of the foreign banks. Thus, even in the relatively simple case with only two countries, defining the exact equity lower limit turns out to be quite complicated, due to cross dependences between countries.

In conclusion one can state that even though the investment diversification reduces the risk of domestic sovereign debt to banks, it may lead to risk contagion between countries. The logic how the credit risk is linked between countries in the two-country model can easily be seen to apply to relationships between multiple countries. In the closely related group of countries like Eurozone, the risk contagion is an especially possible outcome and can have severe economic consequences.

7.3 Risk transition between countries

The two-country model allows studying how cross dependencies between countries affect the occurrence of the feedback loop, but also how the credit risk transfers between countries. Especially interesting results is that the domestic sovereign risk can reflect to domestic banks from their foreign bond holdings regardless of the domestic bond holdings.

The previous chapter presented how the feedback loop can be the result of foreign bond holdings, which shows how the sovereign credit risk transfers from one country to another via banking sector. In the models presented in this paper the repricing depends on the exogenous sunspot probability, which does not contain any information about the state of the economy. This means that a bond repricing in the two-country model may happen also in a country with high fiscal surplus $\overline{S}$. With high surplus repricing does not lead to a feedback loop since government is always able to fully finance the bailout and repay its debt. However, if the other country in the two-country model does not have high surplus, and the banks' equity in that country is below the equity threshold (see equations 36 and 39), repricing creates a feedback loop. So, the credit risk may transfer from one country to another regardless of the economic situation of the country whose bond is repriced. The same logic of course applies when the banking sector in one country is strong, i.e. has enough equity to prevent the feedback loop.

When banks hold foreign bonds, there is a direct connection from foreign sovereign credit risk to domestic banks' balance sheets. A more interesting result from the two-country model is that foreign bond holdings may also contain some part of domestic sovereign credit risk. This is most easily seen when one studies situation where domestic banks do not hold any domestic debt, but the foreign banks hold both bonds. Technically, this means setting the domestic exposure parameter $\alpha = 0$ in the model. Now the equity threshold equation becomes 40. In this equation, domestic bond repricing $\tau\psi L_0$ still exists. So, the equity threshold of domestic banks is affected by the domestic sovereign debt even when the banks do not hold any domestic bonds. What happens here is that domestic bond repricing deteriorates foreign banks' balance sheets, which causes a credit crunch and possibly even a government bailout. The credit crunch and bailout increase the risk of foreign sovereign debt and result in a price drop in the foreign bonds. This foreign bond price drop now deteriorates the domestic banks' balance sheets that contain the foreign bonds, and finally this deterioration leads to a credit crunch and even, in the worst case, to a domestic government bailout. Hence, the domestic sovereign credit risk reflects through the foreign debt to domestic banks, which means that banks are not safe from sovereign debt risk even when they do not directly invest in that debt. It is also worth to point out that the effect of domestic bond repricing $\tau\psi L_0$ in 40 is increasing in β^*, meaning that the larger the foreign banks' exposure the bigger the effect of the risk transition. Again, the foreign banks' investment decisions affect the risk of the domestic banks.

Foreign debt may now contain part of the domestic sovereign risk. This can be called an indirect effect of domestic sovereign credit risk. Existence of this indirect effect means that banks cannot fully protect themselves from the sovereign credit risk with simple portfolio reallocation. This can be formally seen from the general two-country equity threshold equation 39, or in the extreme case from equation 40 as explained above. Decreasing α and increasing β in 39 means allocating investment from domestic to foreign debt. The decrease in α decreases the direct effect of the domestic debt, but increase in β increases the indirect effect of the domestic debt. Thus, the reallocation does not decrease the exposure to domestic debt risk in one to one ratio.

At this point, it is worth noticing that the effect of sovereign risk when it is only present via foreign bond holdings is very limited. For example in equation 40 the domestic credit crunch term is multiplied by both domestic and foreign parameters: foreign exposure β and β^*, inverse of sunspot probability parameters $(1-p)$ and $(1-q)$ and with low surplus parameters π and ρ. Obviously, the effect shrinks to quite small. So at least, it is unlikely that a feedback loop can occur between domestic government and foreign banks, with foreign government being unaffected.

Even a relatively simple two-country model reveals complex cross dependencies in the financial sector. In Chapter 7.2 a symmetric case, when banks hold identical portfolios, was presented. The symmetric case meant the situation where countries together create a pooled asset containing bonds from all the countries and banks are required to hold only that pooled asset (Brunnermeier et al. 2016). This asset can lead to perfect contagion. Also without this pooled asset, the feedback loop can transfer from country to another and risk of one country can transfer via the bonds of other countries even without direct exposure. Moreover, banks in different countries affect each other through their portfolios even though the model presented here does not contain any direct connection between banks. Thus, a single government, not to mention a single bank cannot effectively control all the risk, without extreme manners like capital mobility controls, hence the international coordination and co-operation is required.

7.4 Banking union

The simple analysis of the banking union using the banking union model in Chapter 6 shows, that in this framework, transferring the bailout decision and cost from sovereign to the banking union prevents the feedback loop effectively. This is because the bailout cost does not burden the sovereign anymore, and the sovereign default risk disappears. The banking union does not however remove all the costs from the sovereign, since the credit crunch when banks are distressed still affects the sovereign.

In Eurozone, the ongoing creation of banking union aims to build a resolution fund to cover bailout costs and that fund is financed by banks (European Comission, 2015a). Then the member states would have no direct costs from banking union. In Chapter 6 the cost of banking union membership was imposed on sovereign via credit crunch mechanism. This way the sovereign is not directly involved in payments to the banking union, but faces a cost when banking sector needs to reduce lending in order to pay the membership fees, which affects economic activity and tax revenue.

7.5 Cost of banking union to sovereign

The cost of banking union to sovereign cannot be greater than the cost of bailing out banks by itself. This is the argument behind the model of banking union cost in Chapter 6. The result is formalized in equation 45 and 46, which gives the maximum price of banking union membership and maximum share of lending that this maximum price corresponds. Even though this kind of price limit is relatively artificial, it allows rather reasonable analyses of the issues related to the banking union.

First, the price is increasing in sovereign debt exposure, presented by parameters α and β. This is natural since the bailout becomes more expensive when the exposure increases. Thus, the countries with a banking sector that has large holdings of sovereign debt benefits more from banking union bailouts and can pay a higher price for the membership.

Second, the price is also increasing in the probability of low fiscal surplus, both domestic and foreign, presented by parameters π and ρ. This is interesting, yet logical result since the low surplus leads to the feedback loop equilibrium when sunspot occurs, and increasing costs to government. Thus, the banking union brings more essential security to governments in weak fiscal situation and they are willing to pay a higher price for the membership.

Thirdly, the price is decreasing in banks' aggregate equity E_0, which is an evident result since the higher equity reduces the bailout cost and thus benefits from the banking union. This means that the countries that have banking sector with strong balance sheets are less willing to pay for the banking union membership.

The price is decreasing in sunspot probabilities p and q. The reason again is that the bond prices are calculated in relation to the sunspot probability and thus, the higher risk is more priced into bond values resulting in smaller price drop and bailout cost. More interesting interpretation is that if banks hold a large share of high risk bonds, the country benefits less from banking union, because the bailout cost is smaller when repricing occurs. This can also be interpreted such that, at country level banking union does not increase the risk of a moral hazard. This is a bit daring interpretation, because it is based on the perfect information, i.e. all the agents are aware of the probabilities p and q, hence the bonds are always priced correctly, and excessive risk taking does not really exist in this model.

The share of lending in equation 46 is decreasing in initial lending L_0 and the share of tax revenue from lending τ. This is interesting result, which can be interpreted such that if country's banking sector is large, i.e. large L_0, the smaller share results larger actual losses to the economy. The same logic applies to tax revenue share τ, the higher tax revenue share, the more costly the same banking union share is to the sovereign.

The lending decrease due to payments from banks to the banking union has also another effect to sovereigns in case the bailout is needed. Even though the sovereigns still suffer from the negative effect of a credit crunch when banks face solvency problems, the credit crunch is at least some level smaller due to decreased lending caused by the banking union payments. This can be seen in example from the formalization in equation 42 where the lending is only $1 - \omega$ share of the original.

These results can also be seen from the banking union point of view. Since an increase in the exposure to the sovereign debt increases the bailout costs, the banking union may want to limit this exposure of the banks inside the union. Similarly, the banking union may want to set some requirements for the banks' equity to limit the bailout costs. For the banking union, the high fiscal surplus of its member states is desirable, yet it can be rather difficult for the banking union to conduct fiscal coordination in practice.

7.6 Banking union funds and bailout scenarios

The maximum cost that sovereign is willing to pay for joining the banking union, explained in previous chapter, also gives the limit to the amount of funds that the banking union can obtain. Chapter 6.2 presents some different methods that banking union may use when defining the size of the membership fee. The higher the fee, the more it represses the member states economy. On the other hand, the banking union funds create a limit to bailout costs that the banking union is able to cover. So, there exists a trade-off between the banking union's bailout capacity and the disturbance to the economy of the member state. The following presents different scenarios of bailouts, with varying banking union membership fees.

The first situation is a kind of a benchmark case where the banking union fee is country specific and as high as possible. This gives the absolute maximum amount of funds the banking union can collect from the member states. In this borderline case, the banking union always collects enough funds from a country that it can cover the possible bailout cost of that country's banks. The situation is formalized in equations 65 and 66, in which the term $(1 - \psi)\tau$ is critical to analysis. This term is always less or equal to one, and appears in the funds side of the equation 66, because the banking union does not face the credit crunch, that is why the $(1 - \psi)$ part, and the membership fee only affects the sovereign with respect to tax revenue share, the coefficient τ. This can be generalized such that the bailout is always cheaper or in the extreme case as expensive to the banking union than to the sovereign, which further means that the banking union is always able to collect enough funds to do bailout even in the worst-case scenario, where banks in both countries need to be bailed out simultaneously.

The absolute maximum funds case shows that the banking union is very strong in this model, and can collect as much funds as it needs. However, it may not be optimal to use this maximum membership fee. If the critical term $(1 - \psi)\tau$ in 66 is strictly less than one, the banking union actually collects excessive amount of funds, meaning that even when the banking union bails out banks in both countries, it still has funds left. Since the banking union does not have any other duties than bailouts in this model, it does not need this surplus in anything. This leads directly to the second case, in which the banking union fee is only a fixed fraction of the maximum fee. This is formalized in equations 65 – 69, in which the coefficient μ is the fraction of the maximum fee. In this case, it is directly seen that when $\mu = (1 - \psi)\tau$, the funds collected from a country are equal with the bailout costs of that country's banks. Now, when the banking union bails out the banks in both countries it uses all of its funds and no excess funds are collected. This is then the optimal membership fee, which is just high enough that the banking union always succeeds in bailouts, but creates the minimum disturbance to the sovereigns' economy.

The banking union may well select a membership fee that is lower than the optimal presented above. This clearly means that the simultaneous bailout of the banks in both countries in the model is not possible anymore. It may be argued that the simultaneous bailout case is so rare, that the banking union may take the risk and choose the lower membership fee to spare the member states from the credit crunch effects of the higher fees. However, if the selected membership fee is low enough, e.g. small μ in equation 69, the banking union may be unable to bailout banks in either of the countries. This can be called a total failure of the banking union, and may in the worst case result in the feedback loop in both of the countries. This is however quite unlikely outcome and would require a noticeable shortage of funds in the banking union.

Unlike the total failure, the situation in which the banking union is able to bail out banks in one country, but not in the other, a partial failure, is more likely scenario. A specific case of the partial failure is formalized at the end of chapter 6.3. In this case the whole funds of the banking union are used to cover bailout cost of banks in country one, and the bailout in country two is left for the sovereign. Like shown in equation 80, with realistic parameter values the country two is better off without the banking union membership in this scenario. It is also worth noticing, that in this partial failure situation, the feedback loop is even more likely in country two than before, because the sovereign is burdened by the banking union membership fee on top of everything else. If the partial failure is less dramatic and a part of the bailout cost in country two can also be covered by the banking union, the membership may again be profitable to both countries.

The failure of banking union may result from the membership fee being too low, but also it may result from divergences between the member states of the banking union. The bailout cost in equation 49 is increasing in initial lending L_0 and decreasing in initial equity E_0 and the same applies to funds collected from a country in equation 51. Now, if country one has a large banking sector, i.e. large L_0, with poor solvency situation, i.e. low E_0, and the country two has a small banking sector, with good solvency, the bailout cost of banks in country one is high, but at the same time the funds collected from country two are small. In the extreme case, this may even lead to a total failure of banking union, even when the membership fee is not particularly low.

Chapter 6.2 presents also a formalization of the case in which the banking union fee is set using the same share of lending $\underline{\omega}$ for both of the countries. This solution would probably support the equality principle, but considering the previous scenario of diverging countries, the robustness of this type of membership fee setting is highly questionable. In general, this type of common share would only take into account the different sizes of the banking sectors between countries, while the country specific membership fee also takes into account the country specific risk factors.

The model of banking union presented in this paper is a stylized and heavily simplified formalization from anything that is implemented in reality. Even so, it manages to show that the success of the banking union crucially depends on the design of it, which corresponds the questions presented in other papers like Constâncio (2015) and Béranger and Scialom (2015). If the banking union is too weak, having too little recourses, it may even leave member states worse than without the union. On the other hand, too costly union will depress the economies of member states, resulting yet another suboptimal outcome. Also, the model in this paper suggest that it may be beneficial that the banking union sets some requirements e.g. for banks' equity and sovereign debt exposure and for fiscal surplus of the member states. These suggestions are on a general level in line with the plans of the European banking union, e.g. presented in European Comission (2015a) and European Comission (2015b).

7.7 Comparing results with literature

The one-country model presented in this paper follows the original work of Brunnermeier et al. (2016) and two-country model is extension to it. Brunnermeier et al. also present a two-country model where e.g. sunspot probabilities are independent between countries. The formal two-country model presented in this paper manages to capture and formalize the results of feedback loop contagion that Brunnermeier et al. present. The model presented here diversifies the countries by assigning them independent probabilities for sunspot and fiscal surplus, as well as banks' portfolios are allowed to diverge from each other. This enables studying different scenarios related to portfolio decision as well as the effect of different types of foreign countries. The focus of Brunnermeier et al. (2016) is slightly different, since they focus more on risk transmission via shared assets. Also, the formalization of banking union in this paper is novel innovation and not present in Brunnermeier et al. (2016).

The portfolio structure in the two-country model with foreign and domestic bonds is similar with Farhi and Tirole (2016). However, here the foreign bonds contain risk similarly than domestic bonds while Farhi and Tirole define the foreign bond as a safe asset. Thus, Farhi and Tirole are not presenting any results about the risk transfer between countries, but focus more on the domestic issues like the role of financial supervisor. The supervisor in their model enforces a limit on the risky assets banks can hold, which corresponds the maximum amount of safe assets available to banks solved with the one-country model in Chapter 4.5. In general, this paper focuses more on the equity requirements for banks which Farhi and Tirole do not cover, but in general the focus in their paper is much wider.

Many of the results in this paper regarding the role of banks' equity and its relationship to bailout cost, as well as the mechanism that creates the feedback loop has similarities with many of the other studies like Cooper and Nikolov (2013) or Leonello (2014), but also these studies like Farhi and Tirole (2016) do not present the relationship between domestic banks and foreign credit risk or foreign banks' investments, like the two-country model in this paper.

Acharya et al. (2014) have foreign exposure in their empirical analysis, and interestingly they find statistically significant correlation between foreign debt exposure and domestic banks' credit risk using CDS data, implying the risk transfer between countries via banking sector as suggested by the two-country model presented in this paper. Some similarities with this paper about the role of foreign bond holdings can be found in Uhlig (2014), even though he does not study feedback loop per se, but the role of sovereign and bank credit risk in monetary union with a common central bank. Uhlig finds support for risk-shifting hypothesis, i.e. that banks and sovereigns can shift part of the sovereign debt risk to the common central bank.

Summing up, the academic literature does not contain many formal two-country or multi-country models of the feedback loop. The role of foreign debt and risk transfer is covered in some extent in Brunnermeier et al. (2016), and also Acharya et al. (2014) and Uhlig (2014) make some contribution to the subject. The formal models of the banking union in general, but also in the feedback loop context are even more rare. The two-country model presented in this paper allows a detailed formalization and analysis of the risk transfer, the role of banks' investment decisions and the contagion of the feedback loop between countries. The banking union model presents a stylized formalization of how the banking union prevents the feedback loop, but also reveals the key issues that need to be taken into account in its design.

8 Conclusions

A formal mathematical one- and two-country models of the feedback loop between sovereign and bank credit risks were presented. These models show that the feedback loop can occur when the banks are exposed to the sovereign debt, their equity is low, and a fiscal surplus of the sovereign is low. Under these circumstances, the repricing of the sovereign debt leads to the feedback loop. The indication is that the feedback loop can be avoided by requiring banks to hold enough equity. The models show that this equity threshold depends on the level of exposure to sovereign debt, the probability of low fiscal surplus and inversely on the repricing probability. In the two-country case the equity threshold is additionally affected by the same factors of the foreign debt, but also the equity of foreign banks. This means that already in the two-country scenario defining the exact equity threshold is complicated.

Formal mathematical two- or multi-country models in the feedback loop context are rare in the academic literature. On one hand, this is understandable considering the heavy influence of home bias related to the feedback loop, and the ability of one-country models to describe this situation. On the other, it is surprising because the crisis in Eurozone indicates the risk contagion from country to another. The analysis in this thesis using the two-country model are indeed interesting. The repricing of either one of the bonds that banks hold can cause the feedback loop in one, or the other, or both countries. Especially interesting is the result that banks may be exposed to the risk of their domestic sovereign debt via foreign sovereign debt holdings. This means that for example bank regulations which prevent the direct holdings of the sovereign debt are unable to fully prevent the exposure to the risk of that debt.

As an extension to the two-country model, a formal model of banking union was created in this thesis. Since the European banking union is a relatively recent step in the European integration, only limited amount of academic literature covers the topic. The simple analysis using the banking union model in this thesis not only shows that the banking union breaks the link from banks to sovereign preventing the feedback loop, but also reveals that the design of the banking union is vital to its success. The banking union design faces a trade-off between the banking union's bailout capacity and the disturbance to the economy of the member state. The analysis using the banking union model shows also that it is beneficial to include some regulation in equity requirements and sovereign debt exposures for the banks in the banking union.

In general, situations like the feedback loop, where two phenomena are simultaneously the cause and the effect of each other, are interesting but less covered in economic literature, something that probably changes in the future. Also, the role of sovereign debt and the fragility of banking sector, and especially the questions related to banking unions remain topical issues in economics.

References

Acharya, V., Drechsler, I., and Schnabl, P. (2014). A pyrrhic victory? Bank bailouts and sovereign credit risk. *The Journal of Finance*, 69(6):2689–2739.

Acharya, V. and Steffen, S. (2015). The "greatest" carry trade ever? Understanding eurozone bank risks. *Journal of Financial Economics*, 115(2):215–236.

Aghion, P., Bolton, P., and Fries, S. (1999). Optimal design of bank bailouts: the case of transition economies. *Journal of Institutional and Theoretical Economics (JITE)/Zeitschrift für die gesamte Staatswissenschaft*, pages 51–70.

Basel Committee on Banking Supervision (2006). *Basel II: International convergence of capital measurement and capital standards: A revised framework–comprehensive version*. Bank for International Settlements.

Battistini, N., Pagano, M., and Simonelli, S. (2014). Systemic risk, sovereign yields and bank exposures in the euro crisis. *Economic Policy*, 29(78):203–251.

Béranger, A. and Scialom, L. (2015). Banking union: Mind the gaps. *International Economics*, 144:95–115.

Bianchi, J. (2016). Efficient bailouts? *American Economic Review*, 106(12):3607–59.

BIS Monetary and Economic Department (2013). Low rates spur credit markets as banks lose ground. *BIS Quarterly Review*, (December):1–12.

Bocola, L. (2016). The pass-through of sovereign risk. *Journal of Political Economy*, 124(4):879–926.

Brunnermeier, M. K., Garicano, L., Lane, P. R., Pagano, M., Reis, R., Santos, T., Thesmar, D., Nieuwerburgh, S. V., and Vayanos, D. (2016). The Sovereign-Bank Diabolic Loop and ESBies. *American Economic Review*, 106(5):508–512.

Calvo, G. A. (1988). Servicing the public debt: The role of expectations. *The American*

Economic Review, pages 647–661.

Chari, V., Dovis, A., and Kehoe, P. J. (2014). On the optimality of financial repression. *Manuscript, University of Minnesota*.

Constâncio, V. (2015). Banking union: meaning and implications for the future of banking. *The Spanish Review of Financial Economics*, 1(13):1–6.

Cooper, R. and Nikolov, K. (2013). Government debt and banking fragility: The spreading of strategic uncertainty. Technical report, National Bureau of Economic Research.

Diamond, D. W. and Dybvig, P. H. (1983). Bank runs, deposit insurance, and liquidity. *Journal of political economy*, 91(3):401–419.

Diamond, D. W. and Rajan, R. G. (2002). Bank bailouts and aggregate liquidity. *The American Economic Review*, 92(2):38–41.

European Comission (2015a). Completing the banking union. Online resource retrieved 15.4.2017 from https://ec.europa.eu/info/business-economy-euro/banking-and-finance/banking-union/.

European Comission (2015b). Updated version of first memo published on 15/04/2014 - Banking Union: restoring financial stability in the Eurozone. *European Commission - Fact Sheet*, (MEMO/15/6164).

European Systemic Risk Board (2015). ESRB report on the regulatory treatment of sovereign exposures.

Farhi, E. and Tirole, J. (2016). Deadly Embrace: Sovereign and Financial Balance Sheets Doom Loops. Working Paper 164191, Harvard University OpenScholar.

Gorton, G. and Huang, L. (2004). Liquidity, efficiency, and bank bailouts. *The American Economic Review*, 94(3):455–483.

Hull, J. (2009). *Options, Futures and Other Derivatives*. Pearson Education.

Keister, T. (2016). Bailouts and financial fragility. *The Review of Economic Studies*, 83(2):704–736.

Leonello, A. (2014). Government guarantees and the two-way feedback between banking and sovereign debt crises. *Unpublished working paper*.

Pozzolo, A. F., Calzolari, G., and Navaretti, G. B. (2016). Diabolic loop or incomplete union? sovereign and banking risk. *European Economy – Banks, Regulation, and the Real Sector*, (2016.2).

Reinhart, C. M. and Rogoff, K. S. (2011). The forgotten history of domestic debt. *The Economic Journal*, 121(552):319–350.

Tesar, L. L. and Werner, I. M. (1995). Home bias and high turnover. *Journal of international Money and Finance*, 14(4):467–492.

Uhlig, H. (2014). Sovereign default risk and banks in a monetary union. *German Economic Review*, 15(1):23–41.

Mathematical appendix

One-country model

Derivation of the equation 7 based on the equation 6

$\Delta_1 = \pi[\tau\psi L_0 - \alpha(-(1-p)\Delta_1) - E_0)] \Leftrightarrow$

$\Delta_1 + \pi\alpha(-(1-p)\Delta_1) = \pi[\tau\psi L_0 - E_0)] \Leftrightarrow$

$\Delta_1 = \frac{\pi(\tau\psi L_0 - E_0)}{1-\pi\alpha(1-p)}$

and continuing to equation 11

$\alpha(-(1-p)\Delta_1) + E_0 < 0 \Leftrightarrow$ plugging in Δ_1

$-\alpha(1-p)\frac{\pi(\tau\psi L_0 - E_0)}{1-\pi\alpha(1-p)} + E_0 < 0 \Leftrightarrow$

$-\alpha(1-p)\frac{\pi(\tau\psi L_0 - E_0)}{1-\pi\alpha(1-p)} < -E_0 \Leftrightarrow$

$-\alpha(1-p)\pi(\tau\psi L_0 - E_0) < -[1-\pi\alpha(1-p)]E_0 \Leftrightarrow$

$-\alpha(1-p)\pi(\tau\psi L_0 - E_0) < -[1-\pi\alpha(1-p)]E_0 \Leftrightarrow$

$-\alpha(1-p)\pi\tau\psi L_0 + \alpha(1-p)\pi E_0 < -[1-\pi\alpha(1-p)]E_0 \Leftrightarrow$

$\alpha(1-p)\pi\tau\psi L_0 - \alpha(1-p)\pi E_0 > [1-\pi\alpha(1-p)]E_0 \Leftrightarrow$

$\alpha(1-p)\pi\tau\psi L_0 > \alpha(1-p)\pi E_0 + [1-\pi\alpha(1-p)]E_0 \Leftrightarrow$

$\alpha(1-p)\pi\tau\psi L_0 > [\pi\alpha(1-p) + 1 - \pi\alpha(1-p)]E_0 \Leftrightarrow$

$\alpha(1-p)\pi\tau\psi L_0 > E_0$

Calculating equation 16

$E_0 > \alpha\frac{(1-p)\pi}{1-\alpha(1-p)\pi}(\tau\psi L_0 - E_0) - \tau\psi L_0 \Leftrightarrow$

$(1-\alpha(1-p)\pi)E_0 > \alpha(1-p)\pi(\tau\psi L_0 - E_0) - (1-\alpha(1-p)\pi)\tau\psi L_0 \Leftrightarrow$

$(1-\alpha(1-p)\pi)E_0 > \alpha(1-p)\pi\tau\psi L_0 - \alpha(1-p)\pi E_0 - (1-\alpha(1-p)\pi)\tau\psi L_0 \Leftrightarrow$

$(1-\alpha(1-p)\pi)E_0 + \alpha(1-p)\pi E_0 > \alpha(1-p)\pi\tau\psi L_0 - (1-\alpha(1-p)\pi)\tau\psi L_0 \Leftrightarrow$

$(1-\alpha(1-p)\pi + \alpha(1-p)\pi)E_0 > (\alpha(1-p)\pi - 1 + \alpha(1-p)\pi)\tau\psi L_0 \Leftrightarrow$

$E_0 > (2\alpha(1-p)\pi - 1)\tau\psi L_0$

Two-country model

Simplifying equity threshold equation 35 to 36:

$-\alpha(1-p)\frac{\pi[\tau\psi L_0+\beta(1-q)\Delta_1^*-E_0]}{1-\alpha\pi(1-p)}-\beta(1-q)\Delta_1^*+E_0<0$

$\Leftrightarrow$

$-\alpha(1-p)\pi[\tau\psi L_0+\beta(1-q)\Delta_1^*-E_0]-(1-\alpha\pi(1-p))\beta(1-q)\Delta_1^*$

$<-(1-\alpha\pi(1-p))E_0$

$\Leftrightarrow$

$-\alpha(1-p)\pi\tau\psi L_0-[\alpha(1-p)\pi\beta(1-q)+(1-\alpha\pi(1-p))\beta(1-q)]\Delta_1^*$

$<[-\alpha(1-p)\pi-(1-\alpha\pi(1-p))]E_0$

$\Leftrightarrow$

$\alpha(1-p)\pi\tau\psi L_0+\beta(1-q)\Delta_1^*>E_0:=\underline{E_0}$

Deriving the result presented in equation 38 by combining the following results. Equity threshold of the domestic country is 36:

$\alpha(1-p)\pi\tau\psi L_0+\beta(1-q)\Delta_1^*=\underline{E_0}$

The foreign bond discount is

$\Delta_1^*=\frac{\rho[\tau\psi L_0^*+\beta^*(1-p)\Delta_1-E_0^*]}{1-\alpha^*\rho(1-q)}$

and now foreign banks can hold only their home country's bonds meaning that $\beta^*=0$, thus

$\Delta_1^*=\frac{\rho[\tau\psi L_0^*-E_0^*]}{1-\alpha^*\rho(1-q)}$

Foreign banks meet their equity threshold, meaning that

$\underline{E_0}^*=\alpha^*(1-q)\rho\tau\psi L_0^*+\beta^*(1-p)\Delta_1$

and as defined above $\beta^*=0$ means that

$\underline{E_0}^*=\alpha^*(1-q)\rho\tau\psi L_0^*.$

And plugging this E_0^* to foreign discount results to

$\Delta_1^*=\frac{\rho[\tau\psi L_0^*-E_0^*]}{1-\alpha^*\rho(1-q)}\Leftrightarrow$

$\Delta_1^*=\frac{\rho[\tau\psi L_0^*-\alpha^*(1-q)\rho\tau\psi L_0^*]}{1-\alpha^*\rho(1-q)}\Leftrightarrow$

$\Delta_1^*=\frac{\rho[(1-\alpha^*(1-q)\rho)\tau\psi L_0^*]}{1-\alpha^*\rho(1-q)}\Leftrightarrow$

$\Delta_1^* = \rho\tau\psi L_0^*$.

Now this Δ_1^* can be used in the original equity threshold to get the equation 38:

$\alpha(1-p)\pi\tau\psi L_0 + \beta(1-q)\Delta_1^* = \underline{E_0} \Leftrightarrow$

$\alpha(1-p)\pi\tau\psi L_0 + \beta(1-q)\rho\tau\psi L_0^* = \underline{E_0}$

Derivation of general two-country equity threshold equation 39. First price discounts Δ_1 and Δ_1^* are

$\Delta_1 = \frac{\pi[\tau\psi L_0 + \beta(1-q)\Delta_1^* - E_0]}{1-\alpha\pi(1-p)}$

$\Delta_1^* = \frac{\rho[\tau\psi L_0^* + \beta^*(1-p)\Delta_1 - E_0^*]}{1-\alpha^*\rho(1-q)}$

And equity threshold solved with respect to Δ_1^* is 36 (for derivation see equations 32 - 36):

$\alpha(1-p)\pi\tau\psi L_0 + \beta(1-q)\Delta_1^* = \underline{E_0}$

First, to shorten the notation in calculations denote $A = 1-\alpha\pi(1-p)$ and $B = 1-\alpha^*\rho(1-q)$ and then solve Δ_1^* with respect to Δ_1 results to:

$\Delta_1^* = \frac{\rho[\tau\psi L_0^* + \beta^*(1-p)\Delta_1 - E_0^*]}{1-\alpha^*\rho(1-q)} \Leftrightarrow$

$\Delta_1^* = \frac{\rho[\tau\psi L_0^* + \beta^*(1-p)\Delta_1 - E_0^*]}{B} \Leftrightarrow$

$B\Delta_1^* = \rho[\tau\psi L_0^* + \beta^*(1-p)\Delta_1 - E_0^*] \Leftrightarrow$

$B\Delta_1^* = \rho\tau\psi L_0^* - \rho E_0^* + \rho\beta^*(1-p)\frac{\pi[\tau\psi L_0 + \beta(1-q)\Delta_1^* - E_0]}{1-\alpha\pi(1-p)} \Leftrightarrow$

$B\Delta_1^* = \rho\tau\psi L_0^* - \rho E_0^* + \rho\beta^*(1-p)\frac{\pi[\tau\psi L_0 + \beta(1-q)\Delta_1^* - E_0]}{A} \Leftrightarrow$

$AB\Delta_1^* = A\rho\tau\psi L_0^* - A\rho E_0^* + \rho\beta^*(1-p)\pi[\tau\psi L_0 + \beta(1-q)\Delta_1^* - E_0] \Leftrightarrow$

$AB\Delta_1^* = A\rho\tau\psi L_0^* - A\rho E_0^* + \rho\beta^*(1-p)\pi\tau\psi L_0 + \rho\beta^*(1-p)\pi\beta(1-q)\Delta_1^* - \rho\beta^*(1-p)\pi E_0 \Leftrightarrow$

$AB\Delta_1^* - \rho\beta^*(1-p)\pi\beta(1-q)\Delta_1^* = A\rho\tau\psi L_0^* - A\rho E_0^* + \rho\beta^*(1-p)\pi\tau\psi L_0 - \rho\beta^*(1-p)\pi E_0 \Leftrightarrow$

$(AB - \rho\beta^*(1-p)\pi\beta(1-q))\Delta_1^* = A\rho\tau\psi L_0^* - A\rho E_0^* + \rho\beta^*(1-p)\pi(\tau\psi L_0 - E_0) \Leftrightarrow$

$\Delta_1^* = \frac{A\rho\tau\psi L_0^* - A\rho E_0^* + \rho\beta^*(1-p)\pi(\tau\psi L_0 - E_0)}{(AB - \rho\beta^*(1-p)\pi\beta(1-q))}$

Now to clarify calculation little bit further denote $C = \rho\beta^*(1-p)\pi\beta(1-q))$, so the denominator in Δ_1^* becomes $AB - C$.

Now plugging this Δ_1^* to equity threshold and denoting domestic equity variable as the threshold $\underline{E_0}$, the general equity threshold equation without discount variables Δ_1 and Δ_1^* can be solved:

$\underline{E_0} = \alpha(1-p)\pi\tau\psi L_0 + \beta(1-q)\Delta_1^* \Leftrightarrow$

$\underline{E_0} = \alpha(1-p)\pi\tau\psi L_0 + \beta(1-q)\frac{A\rho\tau\psi L_0^* - A\rho E_0^* + \rho\beta^*(1-p)\pi(\tau\psi L_0 - \underline{E_0})}{(AB-C)} \Leftrightarrow$

$\underline{E_0} = \alpha(1-p)\pi\tau\psi L_0 + \frac{A\beta(1-q)\rho\tau\psi L_0^* - A\beta(1-q)\rho E_0^* + \beta(1-q)\rho\beta^*(1-p)\pi(\tau\psi L_0 - \underline{E_0})}{(AB-C)} \Leftrightarrow$

$\underline{E_0} = \alpha(1-p)\pi\tau\psi L_0 + \frac{A\beta(1-q)\rho\tau\psi L_0^* - A\beta(1-q)\rho E_0^* + C(\tau\psi L_0 - \underline{E_0})}{(AB-C)} \Leftrightarrow$

$\underline{E_0} = \alpha(1-p)\pi\tau\psi L_0 + \frac{A\beta(1-q)\rho\tau\psi L_0^* - A\beta(1-q)\rho E_0^* + C\tau\psi L_0 - C\underline{E_0}}{(AB-C)} \Leftrightarrow$

$(AB-C)\underline{E_0} = (AB-C)\alpha(1-p)\pi\tau\psi L_0 + A\beta(1-q)\rho\tau\psi L_0^* - A\beta(1-q)\rho E_0^* + C\tau\psi L_0 - C\underline{E_0} \Leftrightarrow$

$(AB-C)\underline{E_0} + C\underline{E_0} = (AB-C)\alpha(1-p)\pi\tau\psi L_0 + A\beta(1-q)\rho\tau\psi L_0^* - A\beta(1-q)\rho E_0^* + C\tau\psi L_0 \Leftrightarrow$

$AB\underline{E_0} = (AB-C)\alpha(1-p)\pi\tau\psi L_0 + A\beta(1-q)\rho\tau\psi L_0^* - A\beta(1-q)\rho E_0^* + C\tau\psi L_0 \Leftrightarrow$

$\underline{E_0} = \frac{(AB-C)\alpha(1-p)\pi}{AB}\tau\psi L_0 + (\frac{1}{AB})(A\beta(1-q)\rho\tau\psi L_0^* - A\beta(1-q)\rho E_0^* + C\tau\psi L_0) \Leftrightarrow$

$\underline{E_0} = \frac{(AB-C)\alpha(1-p)\pi}{AB}\tau\psi L_0 + (\frac{1}{AB})(A\beta(1-q)\rho\tau\psi L_0^* - A\beta(1-q)\rho E_0^* + \rho\beta^*(1-p)\pi\beta(1-q)\tau\psi L_0) \Leftrightarrow$

$\underline{E_0} = \frac{(AB-C)\alpha(1-p)\pi}{AB}\tau\psi L_0 + (\frac{1}{AB})(A\beta(1-q)\rho\tau\psi L_0^* - A\beta(1-q)\rho E_0^* + \rho\beta^*(1-p)\pi\beta(1-q)\tau\psi L_0) \Leftrightarrow$

$\underline{E_0} = \frac{(AB-C)\alpha(1-p)\pi}{AB}\tau\psi L_0 + (\frac{\beta(1-q)\rho}{AB})(A\tau\psi L_0^* - AE_0^* + \beta^*(1-p)\pi\tau\psi L_0) \Leftrightarrow$

$\underline{E_0} = \frac{(AB-C)\alpha(1-p)\pi}{AB}\tau\psi L_0 + (\frac{\beta(1-q)\rho}{AB})(A(\tau\psi L_0^* - E_0^*) + \beta^*(1-p)\pi\tau\psi L_0)$

Now by renaming $AB := \Theta$ and $C := \Lambda$, as well as changing $A = 1 - \alpha\pi(1-p) := 1 - A \Rightarrow A = \alpha\pi(1-p)$ the form presented in equation 39 is finished:

$\underline{E_0} = \frac{(\Theta-\Lambda)A}{\Theta}\tau\psi L_0 + \frac{\beta(1-q)\rho}{\Theta}[(1-A)(\tau\psi L_0^* - E_0^*) + \beta^*(1-p)\pi\tau\psi L_0]$

Banking union

Calculating the final result 80 from 79

$\tau\psi L_0^* - (P_1^* - P_0^*) - E_0^* = \tau\psi(1 - \mu\overline{\omega^*})L_0^* - (P_1^* - P_0^*) - E_0^* + \mu\tau\overline{\omega^*}L_0^*$

$\Leftrightarrow$

$\tau\psi L_0^* = \tau\psi(1 - \mu\overline{\omega^*})L_0^* + \mu\tau\overline{\omega^*}L_0^*$

$\Leftrightarrow$

$\psi = \psi - \psi\mu\overline{\omega^*} + \mu\overline{\omega^*}$

$\Leftrightarrow$

$0 = (1 - \psi)\mu\overline{\omega^*}.$